ANALOG COMMUNICATIONS
(20EC0405)

Prepared By-

1. Dr.B.SAROJA Ph.D
Professor
Department Of ECE

2. Mrs.S.ROJA M.Tech
Assistant Professor
Department Of ECE

COURSE OBJECTIVES

The Objective of this course:

1. *To study the fundamental concepts of the analog communication system.*
2. *To analyze various analog modulation and demodulation techniques.*
3. *To know the working of various transmitters and receivers.*
4. *To understand the influence of noise on the performance of analog communication systems, and to acquire the knowledge about information and capacity.*

COURSE OUTCOMES (COs)

On Successful Completion of this Course the Student will be able to

1. *Describe the fundamentals of Analog Communication Systems.*
2. *Express the concept of various Analog Modulation schemes and Multiplexing.*
3. *Compute various parameters of continuous and pulse wave modulation Techniques.*
4. *Analyze various continuous and pulse wave modulation and Demodulation Schemes.*
5. *Estimate the performance of Analog Communication System in the presence of noise.*
6. *Identify different Radio receivers and understand the concept of coding schemes in Information theory.*

SYALLUBUS

UNIT – I

Amplitude Modulation –I: Introduction to Communication Systems – Modulation, Need for Modulation–Introduction to Amplitude Modulation–Power and transmission efficiency, Single tone AM, Generation of AM wave – Square law Modulator & Switching modulator, Detection of AM Wave–Square law detector & Envelope detector, AM Transmitters, Illustrative Problems.

UNIT – II

Amplitude Modulation –II: Introduction to DSB-SC, Power calculations, Generation of DSB-SC, Balanced Modulators& Ring Modulator, Coherent detection of DSB-SC–Time domain description of SSB–Hilbert transform, Generation of SSB wave, Frequency discrimination & Phase discrimination method, Demodulation of SSB Wave–Introduction to Vestigial sideband (VSB)modulation and its Features–Comparison of AM Techniques–Illustrative Problems.

UNIT – III

Angle Modulation: Generalized concept of angle modulation –Frequency modulation, Narrow band frequency modulation (NBFM) and Wide band FM (WBFM), Generation of FM waves, Indirect method, Direct method, Demodulation of FM, Phase modulation – Pre-emphasis& De-emphasis filters – FM Transmitter – Illustrative Problems.

UNIT – IV

Radio Receiver: Introduction to radio receivers & its parameters–Super heterodyne AM & FM Receiver.

Noise: Review of noise and noise sources–noise figure–Performance analysis of AM, DSB-SC, SSB-SC in the presence of noise – Illustrative Problems.

UNIT – V

Analog Pulse Modulation Schemes: Pulse amplitude modulation (PAM) & demodulation, Transmission bandwidth– Pulse-Time Modulation, Pulse Duration and Pulse Position modulations and demodulation schemes– Multiplexing Techniques, FDM, TDM.

Information Theory: Introduction to information theory, Entropy, Mutual information, Channel capacity theorem– Shannon-Fano encoding algorithm–Illustrative Problems.

TEXT BOOKS

1. Simon Haykin, *Communication Systems*,Wiley-India, 2nd Edition, 2010.
2. A. Bruce Carlson, & Paul B. Crilly, *Communication Systems – An Introduction to Signals& Noise in Electrical Communication*,McGraw-Hill, 5th Edition, 2010.

REFERENCES

1. Herbert Taub& Donald L.Schilling,*Principles of Communication Systems*, TataMcGraw-Hill, 3rdEdition, 2009.
2. R.E. Ziemer& W.H. Tranter,*Principles of Communication-Systems Modulation & Noise,* Jaico Publishing House, 2001.
3. George Kennedy and Bernard Davis, *Electronics & Communication System*, TMH, 2004.

UNIT-I
AMPLITUDE MODUATION-I

- Introduction to Communication Systems
- Modulation
- Need for Modulation
- Introduction to Amplitude Modulation
- Power and transmission efficiency
- Single tone AM
- Generation of AM wave
- Square law Modulator
- Switching modulator
- Detection of AM Wave
- Square law detector
- Envelope detector
- AM Transmitters
- Illustrative Problems.

Introduction to Communication System

Communication is the process by which information is exchanged between individuals through a medium.

Communication can also be defined as the transfer of information from one point in space and time to another point.

The basic block diagram of a communication system is as follows.

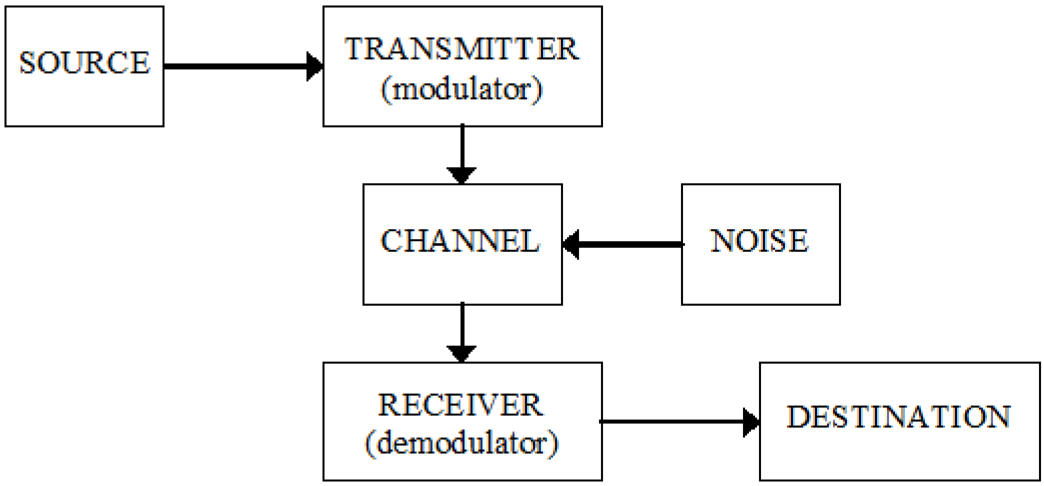


Fig 1.1. Block Diagram Of A Communication System

- **Transmitter:** Couples the message into the channel using high frequency signals.
- **Channel:** The medium used for transmission of signals
- **Modulation:** It is the process of shifting the frequency spectrum of a signal to a frequency range in which more efficient transmission can be achieved.
- **Receiver:** Restores the signal to its original form.
- **Demodulation:** It is the process of shifting the frequency spectrum back to the original baseband frequency range and reconstructing the original form.

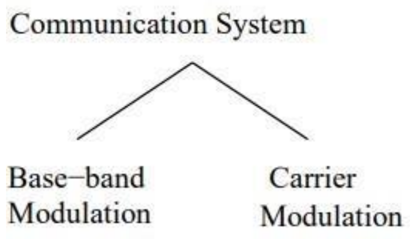
Modulation:

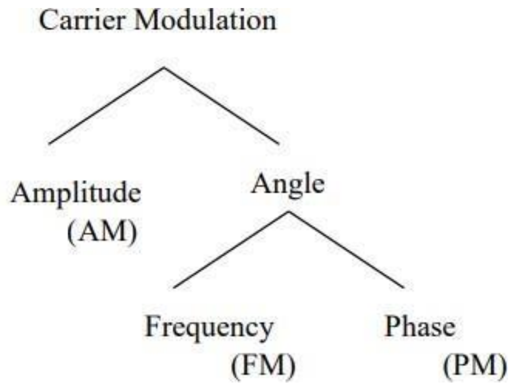
Modulation is a process that causes a shift in the range of frequencies in a signal.

• Signals that occupy the same range of frequencies can be separated.

• Modulation helps in noise immunity, attenuation - depends on the physical medium.

The below figure shows the different kinds of analog modulation schemes that are available





Modulation is operation performed at the transmitter to achieve efficient and reliable information transmission.

For analog modulation, it is frequency translation method caused by changing the appropriate quantity in a carrier signal.

It involves two waveforms:

- A modulating signal/baseband signal – represents the message.
- A carrier signal – depends on type of modulation.

•Once this information is received, the low frequency information must be removed from the high frequency carrier. •This process is known as “Demodulation”.

Need for Modulation:

- Baseband signals are incompatible for direct transmission over the medium so, modulation is used to convey (baseband) signals from one place to another.
- Allows frequency translation:
  - Frequency Multiplexing
  - Reduce the antenna height
  - Avoids mixing of signals
  - Narrow banding
- Efficient transmission
- Reduced noise and interference

Types of Modulation:

Three main types of modulations:

Analog Modulation

- **Amplitude modulation**

Example: Double sideband with carrier (DSB-WC), Double- sideband suppressed carrier (DSB-SC), Single sideband suppressed carrier (SSB-SC), vestigial sideband (VSB)

- **Angle modulation (frequency modulationλ & phase modulation)**

  Example: Narrow band frequency modulation (NBFM), Widebandλ frequency modulation (WBFM), Narrowband phase modulation (NBPM), Wideband phase modulation (NBPM)

Pulse Modulation

- Carrier is a train of pulses
- Example: Pulse Amplitude Modulation (PAM), Pulse width modulation (PWM) , Pulse Position Modulation (PPM)

Digital Modulation

- Modulating signal is analog
  - Example: Pulse Code Modulation (PCM), Delta Modulationλ (DM), Adaptive Delta Modulation (ADM), Differential Pulse Code Modulation (DPCM), Adaptive Differential Pulse Code Modulation (ADPCM) etc.
- Modulating signal is digital (binary modulation)

Example: Amplitude shift keying (ASK), frequency Shift Keyingλ (FSK), Phase Shift Keying (PSK)

Amplitude Modulation (AM)

Amplitude Modulation is the process of changing the amplitude of a relatively high frequency carrier signal in accordance with the amplitude of the modulating signal (Information).

The carrier amplitude varied linearly by the modulating signal which usually consists of a range of audio frequencies. The frequency of the carrier is not affected.

- Application of AM - Radio broadcasting, TV pictures (video), facsimile transmission
- Frequency range for AM - 535 kHz – 1600 kHz
- Bandwidth - 10 kHz

Various forms of Amplitude Modulation

• Conventional Amplitude Modulation (Alternatively known as Full AM or Double Sideband Large carrier modulation (DSBLC) /Double Sideband Full Carrier (DSBFC)

• Double Sideband Suppressed carrier (DSBSC) modulation

• Single Sideband (SSB) modulation

• Vestigial Sideband (VSB) modulation

Time Domain and Frequency Domain Description

It is the process where, the amplitude of the carrier is varied proportional to that of the message signal.

Let m (t) be the base-band signal, m (t) ←→ M (ω) and c (t) be the carrier, $c(t) = A_c \cos(\omega_c t)$. fc is chosen such that fc >> W, where W is the maximum frequency component of m(t). The amplitude modulated signal is given by

$$s(t) = Ac\,[1 + k_a m(t)] \cos(\omega c t)$$

Fourier Transform on both sides of the above equation

$$S(\omega) = \pi\, Ac/2\, (\delta(\omega - \omega c) + \delta(\omega + \omega c)) + k_a Ac/\,2\, (M(\omega - \omega c) + M(\omega + \omega c))$$

k_a is a constant called amplitude sensitivity.

$k_a m(t) < 1$ and it indicates percentage modulation.

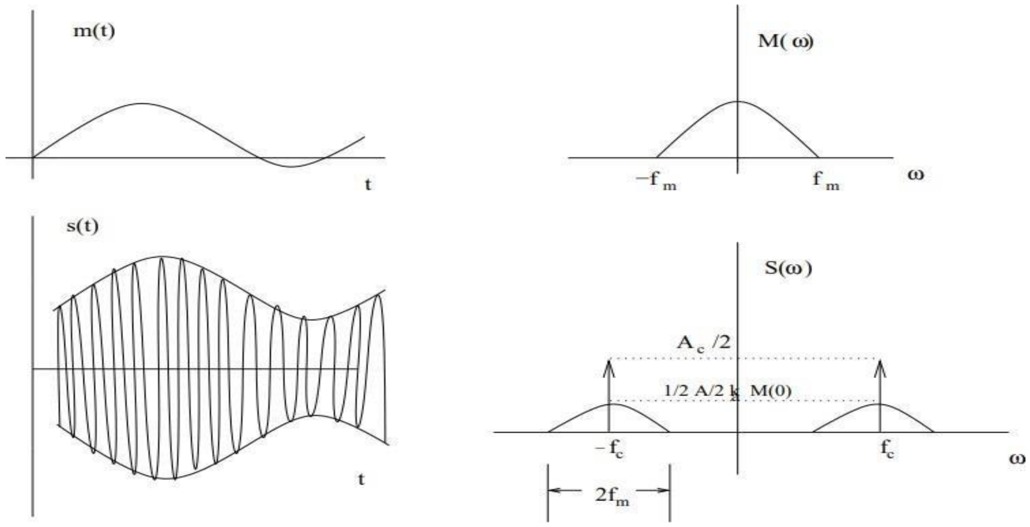


Fig.1.2. Amplitude modulation in time and frequency domain

Power relations in AM waves:

Consider the expression for single tone/sinusoidal AM wave

$$s(t) = A_c Cos(2\pi f_c t) + \frac{1}{2} mA_c Cos[2\pi(f_c + f_m)t] + \frac{1}{2} mA_c Cos[2\pi(f_c - f_m)t] \quad \text{.................(1)}$$

This expression contains three components. They are carrier component, upper side band and lower side band. Therefore Average power of the AM wave is sum of these three components.

Therefore the total power in the amplitude modulated wave is given by

$$Pt = \frac{V^2{}_{car}}{R} + \frac{V^2{}_{LSB}}{R} + \frac{V^2{}_{USB}}{R} \quad \text{........................(2)}$$

Where all the voltages are rms values and R is the resistance, in which the power is dissipated.

$$P_C = \frac{V^2{}_{car}}{R} = \frac{\left(A_c/\sqrt{2}\right)^2}{R} = \frac{A_c{}^2}{2R}$$

$$P_{LSB} = \frac{V_{LSB}{}^2}{R} = \left(\frac{mA_c}{2\sqrt{2}}\right)^2 \frac{1}{R} = \frac{m^2 A_c{}^2}{8R} = \frac{m^2}{4} P_c$$

$$P_{USB} = \frac{V_{USB}{}^2}{R} = \left(\frac{mA_c}{2\sqrt{2}}\right)^2 \frac{1}{R} = \frac{m^2 A_c{}^2}{8R} = \frac{m^2}{4} P_c$$

Therefore total average power is given by

$$P_t = P_c + P_{LSB} + P_{USB}$$

$$P_t = P_c + \frac{m^2}{4}P_c + \frac{m^2}{4}P_c$$

$$P_t = P_c\left(1 + \frac{m^2}{4} + \frac{m^2}{4}\right)$$

$$P_t = P_c\left(1 + \frac{m^2}{2}\right) \qquad \text{.............................(3)}$$

Transmission efficiency:

The ratio of total side band power to the total power in the modulated wave is given by

$$\frac{P_{SB}}{P_t} = \frac{P_c\left(m^2/2\right)}{P_c\left(1 + m^2/2\right)}$$

$$\frac{P_{SB}}{P_t} = \frac{m^2}{2 + m^2} \qquad \text{.........................(4)}$$

This ratio is called the efficiency of AM system

Single Tone Amplitude Modulation:

Consider a modulating wave m(t) that consists of a single tone or single frequency component given by

$$m(t) = A_m \cos(2\pi f_m t) \qquad \text{......................(1)}$$

where A_m is peak amplitude of the sinusoidal modulating wave

f_m is the frequency of the sinusoidal modulating wave

Let A_c be the peak amplitude and f_c be the frequency of the high frequency carrier signal. Then the corresponding single-tone AM wave is given by

$$s(t) = A_c\left[1 + m\cos(2\pi f_m t)\right]Cos(2\pi f_c t) \qquad \text{.........................(2)}$$

Let $A_{\max}$ and $A_{\min}$ denote the maximum and minimum values of the envelope of the modulated wave. Then from the above equation (2.12), we get

$$\frac{A_{\max}}{A_{\min}} = \frac{A_c(1+m)}{A_c(1-m)}$$

$$m = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

Expanding the equation (2), we get

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} m A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2} m A_c \cos[2\pi(f_c - f_m)t]$$

The Fourier transform of $s(t)$ is obtained as follows.

$$s(f) = \frac{1}{2} A_c [\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{4} m A_c [\delta(f - f_c - f_m) + \delta(f + f_c + f_m)]$$

$$+ \frac{1}{4} m A_c [\delta(f - f_c + f_m) + \delta(f + f_c - f_m)]$$

Thus the spectrum of an AM wave, for the special case of sinusoidal modulation consists of delta functions at $\pm f_c$, $f_c \pm f_m$, and $-f_c \pm f_m$. The spectrum for positive frequencies is as shown in figure

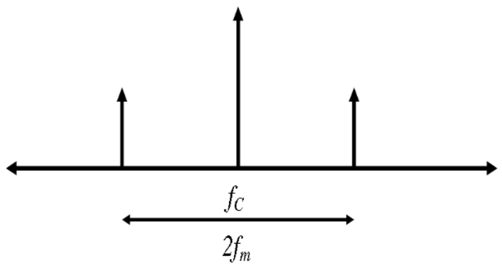


Fig.1.3. Frequency Domain characteristics of single tone AM

Generation of AM waves:

Two basic amplitude modulation principles are discussed. They are square law modulation and switching modulator.

Square Law Modulator

When the output of a device is not directly proportional to input throughout the operation, the device is said to be non-linear. The Input-Output relation of a non-linear device can be expressed as

$$V_0 = a_0 + a_1 V_{in} + a_2 V_{in}^2 + a_3 V_{in}^3 + a_4 V_{in}^4 + ...$$

When the in put is very small, the higher power terms can be neglected. Hence the output is approximately given by $V_0 = a_O + a_1 V_{in} + a_2 V_{in}^2$

When the output is considered up to square of the input, the device is called a square law device and the square law modulator is as shown in the figure 1.4

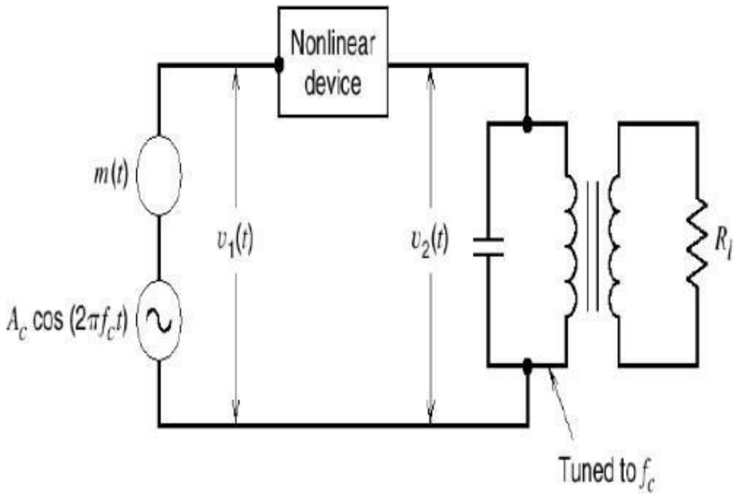


Fig1.4. Square Law Modulator

$$V_{in} = c(t) + m(t)$$
$$V_{in} = A_c \cos 2\pi f_c t + m(t)$$

As the level of the input is very small, the output can be considered up to square of the input, *i.e.*, $V_0 = a_O + a_1 V_{in} + a_2 V_{in}^2$

$$V_0 = a_0 + a_1[A_c \cos 2\pi f_c t + m(t)] + a_2[A_c \cos 2\pi f_c t + m(t)]^2$$

$$V_0 = a_0 + a_1 A_c \cos 2\pi f_c t + a_1 m(t) + \frac{a_2 A_c^2}{2}(1 + \cos 4\pi f_c t) + a_2[m(t)]^2 + 2a_2 m(t) A_c \cos 2\pi f_c t$$

$$V_0 = a_0 + a_1 A_c \cos 2\pi f_c t + a_1 m(t) + \frac{a_2 A_c^2}{2}\cos 4\pi f_c t + a_2 m^2(t) + 2a_2 m(t) A_c \cos 2\pi f_c t$$

Taking fourier transform on both sides

$$V_0(f) = (a_0 + \frac{a_2 A_c^2}{2})\delta(f) + \frac{a_1 A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + a_1 M(f) +$$
$$\frac{a_2 A_c^2}{4}[\delta(f - 2f_c) + \delta(f + 2f_c)] + a_2 M(f) + a_2 A_c[M(f - f_c) + M(f + f_c)]$$

Consider a non-linear device to which a carrier $c(t)=A_c\cos(2\pi f_c t)$ and an information signal m(t) are fed simultaneously as shown in figure1.4. The total input to the deviceat any instant is

Therefore the square law device output 0 V consists of the dc component at f = 0. The information signal ranging from 0 to W Hz and its second harmonics are signal at f_c and $2f_c$.

Frequency band centered at f_c with a deviation of $\pm W$, Hz.

The required AM signal with a carrier frequency f_c can be separated using a band pass filter at the out put of the square law device. The filter should have a lower cut-off frequency ranging between 2W and (f_c-W) and upper cut-off frequency between (f_c+W) and 2 f_c

Therefore the filter out put is

$$s(t) = a_1 A_c \cos 2\pi f_c t + 2a_2 A_c m(t) \cos 2\pi f_c t$$

$$s(t) = a_1 A_c \left[1 + 2\frac{a_2}{a_1} m(t)\right] \cos 2\pi f_c t$$

If $m(t) = A_m \cos 2\pi f_m t$, we get

$$s(t) = a_1 A_c \left[1 + 2\frac{a_2}{a_1} A_m \cos 2\pi f_m t\right] \cos 2\pi f_c t$$

Comparing this with the standard representation of AM signal,

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

Therefore modulation index of the output signal is given by

$$m = 2\frac{a_2}{a_1} A_m$$

The output AM signal is free from distortion and attenuation only when ($f_c - W$)> 2W or $f_c > 3W$.

Spectrum is as shown below

Switching Modulator

Consider a semiconductor diode used as an ideal switch to which the carrier signal $c(t) = A_c \cos(2\pi f_c t)$ and information signal $m(t)$ are applied simultaneously as shown figure

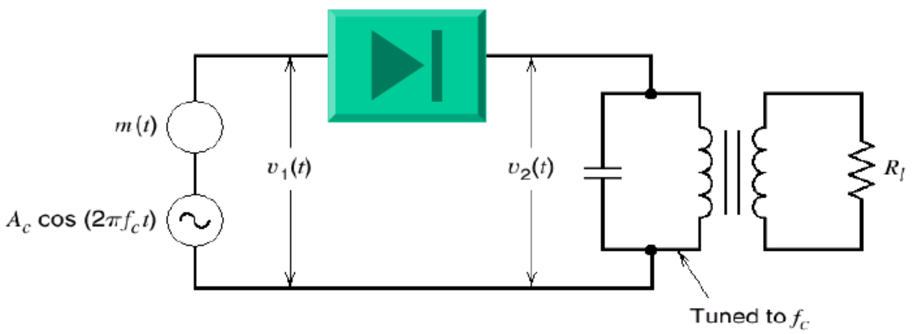


Fig.1.5. Switching Modulator

The total input for the diode at any instant is given by

$$v_1 = c(t) + m(t)$$

$$v_1 = A_c \cos 2\pi f_c t + m(t)$$

When the peak amplitude of c(t) is maintained more than that of information signal, the operation is assumed to be dependent on only c(t) irrespective of m(t).
When c(t) is positive, v2=v1since the diode is forward biased. Similarly, whenc(t) is negative, v2=0 since diode is reverse biased. Based upon above operation, switching response of the diode is periodic rectangular wave with an amplitude unity and is given by

$$p(t) = \frac{1}{2} + \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c t(2n-1))$$

$$p(t) = \frac{1}{2} + \underbrace{\frac{2}{\pi} \cos(2\pi f_c t)}_{n=0,1} - \underbrace{\frac{2}{3\pi} \cos(6\pi f_c t)}_{n=-1,+2} + -$$

Therefore the diode response V_o is a product of switching response *p(t)* and input v_1.

$$v_2 = v_1 * p(t)$$

$$V_2 = [A_c \cos 2\pi f_c t + m(t)] \left[\frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t - \frac{2}{3\pi} \cos 6\pi f_c t + - + - \right]$$

Applying the Fourier Transform, we get

$$V_2(f) = \frac{A_c}{4} [\delta(f - f_c) + \delta(f + f_c)] + \frac{M(f)}{2} + \frac{A_c}{\pi} \delta(f)$$
$$+ \frac{A_c}{2\pi} [\delta(f - 2f_c) + \delta(f + 2f_c)] + \frac{1}{\pi} [M(f - f_c) + M(f + f_c)]$$
$$- \frac{A_c}{6\pi} [\delta(f - 4f_c) + \delta(f + 4f_c)] - \frac{A_c}{3\pi} [\delta(f - 2f_c) + \delta(f + 2f_c)]$$
$$- \frac{1}{3\pi} [M(f - 3f_c) + M(f + f_c)]$$

The diode output v_2 consists of

a dc component at $f = 0$.

Information signal ranging from 0 to *w* Hz and infinite number of frequency bands centered at f, $2f_c$, $3f_c$, $4f_c$, ---------

The required AM signal centred at fc can be separated using band pass filter. The lower cut off-frequency for the band pass filter should be between w and fc-w and the upper cut-off frequency between fc+w and 2fc. The filter output is given by the equation

$$S(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi}\frac{m(t)}{A_c}\right]\cos 2\pi f_c t$$

For a single tone information, let $m(t) = A_m \cos(2\pi f_m t)$

$$S(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi}\frac{A_m}{A_c}\cos 2\pi f_m t\right]\cos 2\pi f_c t$$

Therefore modulation index, $m = \frac{4}{\pi}\frac{A_m}{A_c}$

The output AM signal is free from distortions and attenuations only when $fc\text{-}w > w$ or $fc > 2w$.

Detection of AM waves

Demodulation is the process of recovering the information signal (base band) from the incoming modulated signal at the receiver. There are two methods; they are Square law Detector and Envelope Detector.

Square Law Detector

Consider a non-linear device to which the AM signal s(t) is applied. When the level of s(t) is very small, output can be considered up to square of the input.

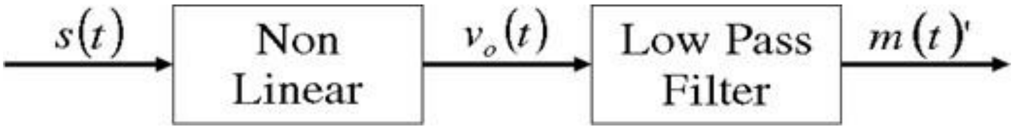


Fig 1.6: Demodulation of AM wave

Therefore $V_o = a_o + a_1 V_{in} + a_2 V_{in}^2$

If *m(t)* is the information signal (0-wHz) and $c(t) = A_c \cos(2\pi f_c t)$ is the carrier, input AM signal to the non-linear device is given by

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

$$V_o = a_o + a_1 s(t) + a_2 [s(t)]^2$$

$$V_o = a_o + a_1 A_c \cos 2\pi f_c t + a_1 A_c K_a m(t) \cos 2\pi f_c t + a_2 [A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t]^2$$

Applying Fourier transform on both sides, we get

$$V_o(f) = \left[a_o + \frac{a_2 A_c^{\ 2}}{2} \right] \delta(f) + \frac{a_1 A_c}{2} [\delta(f - f_c) + \delta(f + f_c)]$$

$$+ \frac{a_1 A_c K_a}{2} [M(f - f_c) + M(f + f_c)] + \frac{a_2 A_c^{\ 2} K_a^{\ 2}}{4} [M(f - 2f_c) + M(f + 2f_c)]$$

$$+ \frac{a_2 A_c^{\ 2} K_a^{\ 2}}{2} \left[\underset{\pm 2W}{M(f)} \right] + \frac{a_2 A_c^{\ 2} K_a^{\ 2}}{2} [M(f - 2f_c) + M(f + 2f_c)]$$

$$+ \frac{a_2 A_c^{\ 2}}{4} [\delta(f - 2f_c) + \delta(f + 2f_c)] + a_2 A_c^{\ 2} K_a [M(f)]$$

The device output consists of a dc component at f =0, information signal ranging from 0-W Hz and its second harmonics and frequency bands centered at fc and 2fc. The required information can be separated using low pass filter with cut off frequency ranging between W and fc-w. The filter output is given by

$$m'(t) = \left(a_o + \frac{a_2 A_c^{\ 2}}{2} \right) + a_2 A_c^{\ 2} K_a m(t) + \frac{a_2 A_c^{\ 2} K_a^{\ 2} m^2(t)}{2}$$

DC component + message signal + second harmonic

The dc component (first term) can be eliminated using a coupling capacitor or a transformer. The effect of second harmonics of information signal can be reduced by maintaining its level very low. When *m(t)* is very low, the filter output is given by

$$m^1(t) = a_2 A_c^{\ 2} K_a m(t)$$

When the information level is very low, the noise effect increases at the receiver, hence the system clarity is very low using square law demodulator.

Envelope Detector

It is a simple and highly effective system. This method is used in most of the commercial AM radio receivers. An envelope detector is as shown below.

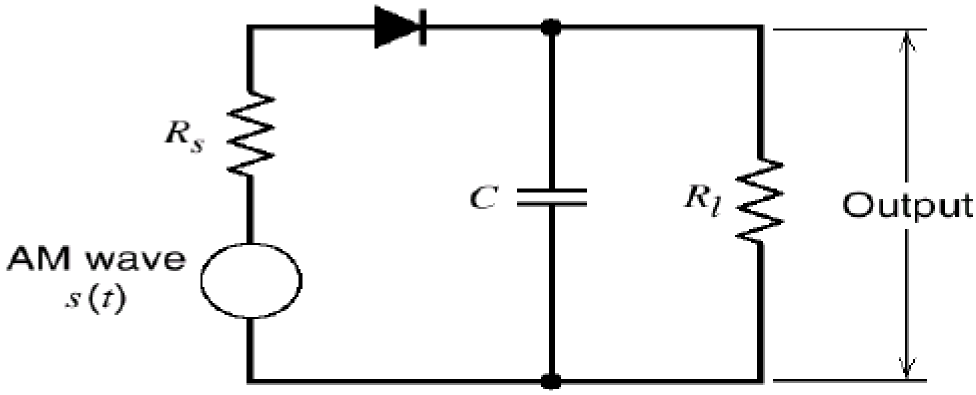


Fig.1.7. Envelope Detector

During the positive half cycles of the input signals, the diode D is forward biased and the capacitor C charges up rapidly to the peak of the input signal. When the input signal falls below this value, the diode becomes reverse biased and the capacitor C discharges through the load resistor RL.

The discharge process continues until the next positive half cycle. When the input signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated.

The charge time constant (rf+Rs)C must be short compared with the carrier period, the capacitor charges rapidly and there by follows the applied voltage up to the positive peak when the diode is conducting.That is the charging time constant shall satisfy the condition,

$$(r_f + R_s)C << \frac{1}{f_c}$$

On the other hand, the discharging time-constant $R_L C$ must be long enough to ensure that the capacitor discharges slowly through the load resistor R_L between the positive peaks of the carrier wave, but not so long that the capacitor voltage will not discharge at the maximum rate of change of the modulating wave.

That is the discharge time constant shall satisfy the condition,

$$\frac{1}{f_c} << R_L C << \frac{1}{W}$$

Where ‘W’ is bandwidth of the message signal. The result is that the capacitor voltage or detector output is nearly the same as the envelope of AM wave.

Advantages and Disadvantages of AM:

Advantages of AM:

- Generation and demodulation of AM wave are easy.
- AM systems are cost effective and easy to build.

Disadvantages:

AM contains unwanted carrier component, hence it requires more transmission power.

- The transmission bandwidth is equal to twice the message bandwidth.

To overcome these limitations, the conventional AM system is modified at the cost of increased system complexity. Therefore, three types of modified AM systems are discussed.

DSBSC (Double Side Band Suppressed Carrier) modulation: In DSBC modulation, the modulated wave consists of only the upper and lower side bands. Transmitted power is saved through the suppression of the carrier wave, but the channel bandwidth requirement is the same as before.

SSBSC (Single Side Band Suppressed Carrier) modulation: The SSBSC modulated wave consists of only the upper side band or lower side band. SSBSC is suited for transmission of voice signals. It is an optimum form of modulation in that it requires the minimum transmission power and minimum channel band width. Disadvantage is increased cost and complexity.

VSB (Vestigial Side Band) modulation: In VSB, one side band is completely passed and just a trace or vestige of the other side band is retained. The required channel bandwidth is therefore in excess of the message bandwidth by an amount equal to the width of the vestigial side band. This method is suitable for the transmission of wide band signals

AM Transmitters

There are two approaches in generating an AM signal. These are known as low and high level modulation. They're easy to identify: A low level AM transmitter performs the process of modulation near the beginning of the transmitter. A high level transmitter performs the modulation step last, at the last or "final" amplifier stage in the transmitter. Each method has advantages and disadvantages, and both are in common use.

Low-Level AM Transmitter:

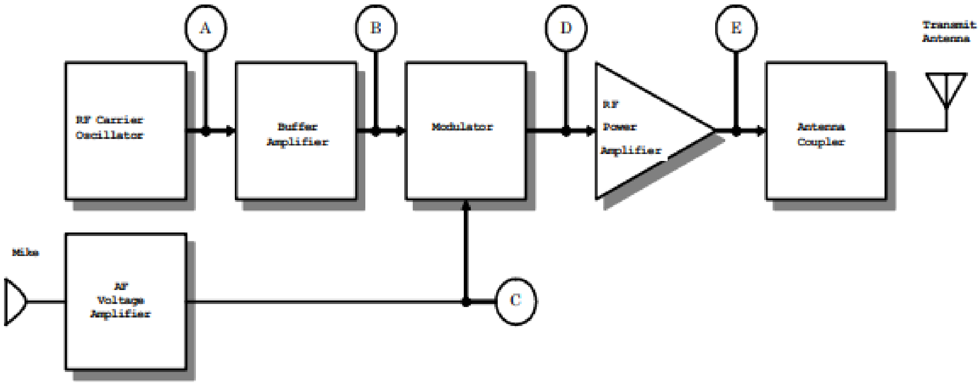


Fig.1.8. Low-Level AM Transmitter Block Diagram

There are two signal paths in the transmitter, audio frequency (AF) and radio frequency (RF). The RF signal is created in the RF carrier oscillator. At test point A the oscillator's output signal is present. The output of the carrier oscillator is a fairly small AC voltage, perhaps 200 to 400 mV RMS. The oscillator is a critical stage in any transmitter. It must produce an accurate and steady frequency. Every radio station is assigned a different carrier frequency. The dial (or display) of a receiver displays the carrier frequency. If the oscillator drifts off frequency, the receiver will be unable to receive the transmitted signal without being readjusted. Worse yet, if the oscillator drifts onto the frequency being used by another radio station, interference will occur. Two circuit techniques are commonly used to stabilize the oscillator, buffering and voltage regulation.

The buffer amplifier has something to do with buffering or protecting the oscillator. An oscillator is a little like an engine (with the speed of the engine being similar to the oscillator's frequency). If the load on the engine is increased (the engine is asked to do more work), the engine will respond by slowing down. An oscillator acts in a very similar fashion. If the current drawn from the oscillator's output is increased or decreased, the oscillator may speed up or slow down slightly.

Buffer amplifier is a relatively low-gain amplifier that follows the oscillator. It has a constant input impedance (resistance). Therefore, it always draws the same amount of current from the oscillator. This helps to prevent "pulling" of the oscillator frequency. The buffer amplifier is needed because of what's happening "downstream" of the oscillator. Right after this stage is the modulator. Because the modulator is a nonlinear amplifier, it may not have a constant input resistance -- especially when information is passing into it. But since there is a buffer amplifier between the oscillator and modulator, the oscillator sees a steady load resistance, regardless of what the modulator stage is doing.

Voltage Regulation: An oscillator can also be pulled off frequency if its power supply voltage isn't held constant. In most transmitters, the supply voltage to the oscillator is regulated at a constant value. The regulated voltage value is often between 5 and 9 volts; zener diodes and three-terminal regulator ICs are commonly used voltage regulators. Voltage regulation is especially important when a transmitter is being powered by batteries or an automobile's electrical system. As a battery discharges, its terminal voltage falls. The DC supply voltage in a car can be anywhere between 12 and 16 volts, depending on engine RPM and other electrical load conditions within the vehicle.

Modulator: The stabilized RF carrier signal feeds one input of the modulator stage. The modulator is a variable-gain (nonlinear) amplifier. To work, it must have an RF carrier signal and an AF information signal. In a low-level transmitter, the power levels are low in the oscillator, buffer, and modulator stages; typically, the modulator output is around 10 mW (700 mV RMS into 50 ohms) or less.

AF Voltage Amplifier: In order for the modulator to function, it needs an information signal. A microphone is one way of developing the intelligence signal, however, it only produces a few millivolts of signal. This simply isn't enough to operate the modulator, so a voltage amplifier is used to boost the microphone's signal. The signal level at the output of the AF voltage amplifier is usually at least 1 volt RMS; it is highly dependent upon the transmitter's design. Notice that the AF amplifier in the transmitter is only providing a voltage gain, and not necessarily a current gain for the microphone's signal. The power levels are quite small at the output of this amplifier; a few mW at best.

RF Power Amplifier: At test point D the modulator has created an AM signal by impressing the information signal from test point C onto the stabilized carrier signal from test point B at the buffer amplifier output. This signal (test point D) is a complete AM signal, but has only a few milli watts of power. The RF power amplifier is normally built with several stages. These stages increase both the voltage and current of the AM signal. We say that power amplification occurs when a circuit provides a current gain. In order to accurately amplify the tiny AM signal from the modulator, the RF power amplifier stages must be linear. You might recall that amplifiers are divided up into "classes," according to the conduction angle of the active device within. Class A and class B amplifiers are considered to be linear amplifiers, so the RF power amplifier stages will normally be constructed using one or both of these type of amplifiers. Therefore, the signal at test point E looks just like that of test point D; it's just much bigger in voltage and current.

Antenna Coupler: The antenna coupler is usually part of the last or final RF power amplifier, and as such, is not really a separate active stage. It performs no amplification, and has no active devices. It performs two important jobs: Impedance matching and filtering. For an RF power amplifier to function correctly, it must be supplied with a load resistance equal to that for which it was designed.

The antenna coupler also acts as a low-pass filter. This filtering reduces the amplitude of harmonic energies that may be present in the power amplifier's output. (All amplifiers generate harmonic distortion, even "linear" ones.) For example, the transmitter may be tuned to operate on 1000 kHz. Because of small nonlinearities in the amplifiers of the transmitter, the transmitter will also produce harmonic energies on 2000 kHz (2nd harmonic), 3000 kHz (3rd harmonic), and so on. Because a low-pass filter passes the fundamental frequency (1000 kHz) and rejects the harmonics, we say that harmonic attenuation has taken place.

High-Level AM Transmitter:

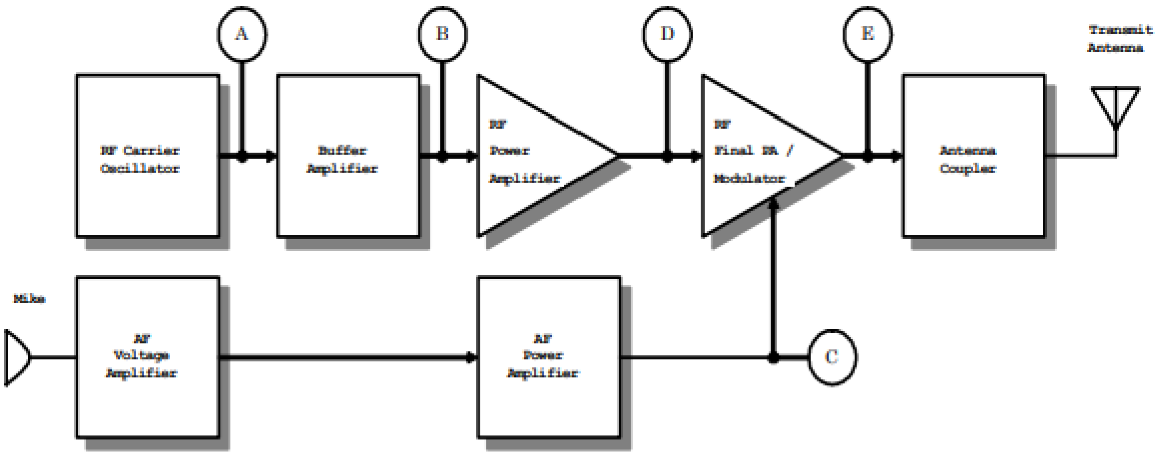


Fig.1.9. Low-Level AM Transmitter Block Diagram

The high-level transmitter of Figure 1.9 is very similar to the low-level unit. The RF section begins just like the low-level transmitter; there is an oscillator and buffer amplifier. The difference in the high level transmitter is where the modulation takes place. Instead of adding modulation immediately after buffering, this type of transmitter amplifies the unmodulated RF carrier signal first. Thus, the signals at points A, B, and D in Figure 9 all look like unmodulated RF carrier waves. The only difference is that they become bigger in voltage and current as they approach test point D.

The modulation process in a high-level transmitter takes place in the last or final power amplifier. Because of this, an additional audio amplifier section is needed. In order to modulate an amplifier that is running at power levels of several watts (or more), comparable power levels of information are required. Thus, an audio power amplifier is required. The final power amplifier does double-duty in a high-level transmitter. First, it provides power gain for the RF carrier signal, just like the RF power amplifier did in the low-level transmitter. In addition to providing power gain, the final PA also performs the task of modulation. The final power amplifier in a high-level transmitter usually operates in class C, which is a highly nonlinear amplifier class.

Comparison:

Low Level Transmitters

- Can produce any kind of modulation; AM, FM, or PM.
- Require linear RF power amplifiers, which reduce DC efficiency and increases production costs.

High Level Transmitters

- Have better DC efficiency than low-level transmitters, and are very well suited for battery operation.Are restricted to generating AM modulation only.

UNIT-II
AMPLITUDE MODULATION-II

- Introduction to DSB-SC
- Power calculations
- Generation of DSB-SC
- Balanced Modulators
- Ring Modulator
- Coherent detection of DSB-SC
- Time domain description of SSB–Hilbert transform
- Generation of SSB wave
- Frequency discrimination
- Phase discrimination method
- Demodulation of SSB Wave
- Introduction to Vestigial sideband (VSB)modulation and its Features
- Comparison of AM Techniques
- Illustrative Problems.

Introduction to DSB-SC:
Time domain and Frequency domain Description:

DSBSC modulators make use of the multiplying action in which the modulating signal multiplies the carrier wave. In this system, the carrier component is eliminated and both upper and lower side bands are transmitted. As the carrier component is suppressed, the power required for transmission is less than that of AM.

If $m(t)$ is the message signal and $c(t) = A_c \cos(2\pi f_c t)$ is the carrier signal, then DSBSC modulated wave $s(t)$ is given by

$$s(t) = c(t)\, m(t)$$

$$s(t) = A_c \cos(2\pi f_c t) m(t)$$

Consequently, the modulated signal s(t) under goes a phase reversal , whenever the message signal m(t) crosses zero as shown below.

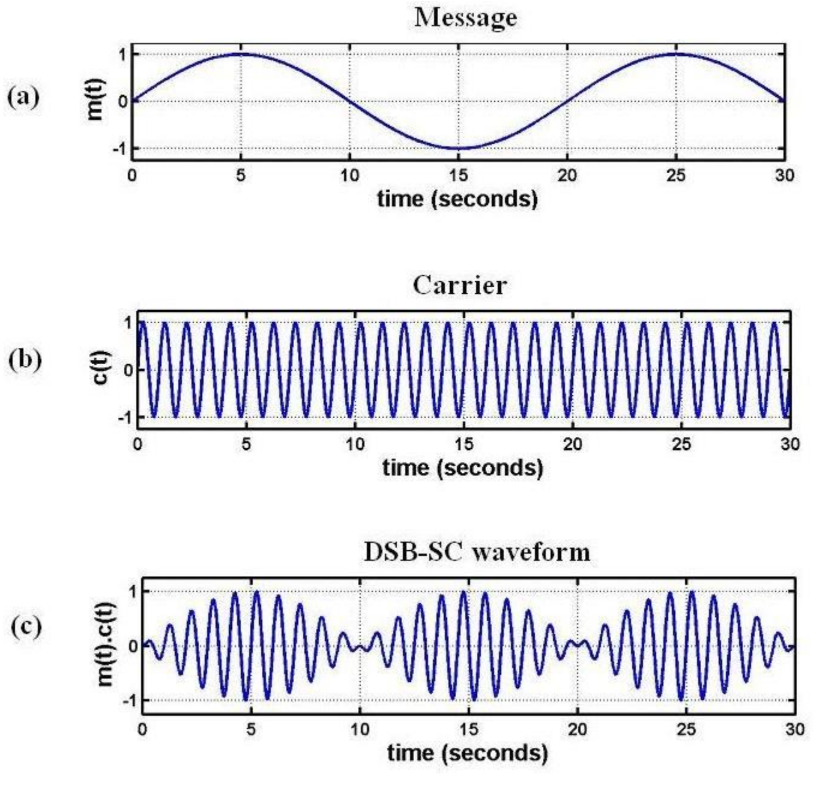


Fig.2.1. (a) DSB-SC waveform (b) DSB-SC Frequency Spectrum

The envelope of a DSBSC modulated signal is therefore different from the message signal and the Fourier transform of s(t) is given by

$$S(f) = \frac{A_c}{2}\left[M(f - f_c) + M(f + f_c)\right]$$

For the case when base band signal *m(t)* is limited to the interval $-W<f<W$ as shown in figure below, we find that the spectrum *S(f)* of the DSBSC wave *s(t)* is as illustrated below. Except for a change in scaling factor, the modulation process simply translates the spectrum of the base band signal by f_c. The transmission bandwidth required by DSBSC modulation is the same as that for AM.

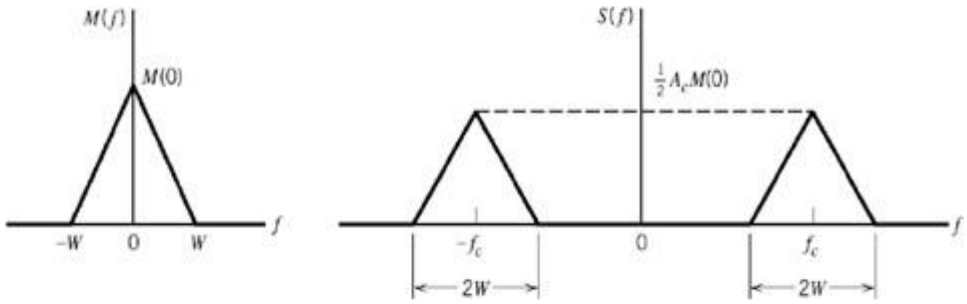


Fig 2.2: Message and DSB-SC Waveforms

POWER CALCUATIONS;

Power of DSB ;

$$P_t = P_{SB} + P_c$$

(X)

As it only contains sidebands.

$$P_t = P_{USB} + P_{LSB}.$$

$$P_{USB} = \left(\frac{A_c A_m}{2}\right)^2 / 2R = \frac{A_c^2 A_m^2}{8R} = P_{LSB}$$

$$\boxed{P_t = \frac{A_c^2 A_m^2}{4R}}$$

Generation of DSBSC Waves:

Balanced Modulator

A balanced modulator consists of two standard amplitude modulators arranged in a balanced configuration so as to suppress the carrier wave as shown in the following block diagram. It is assumed that the AM modulators are identical, except for the sign reversal of the modulating wave applied to the input of one of them. Thus, the output of the two modulators may be expressed as,

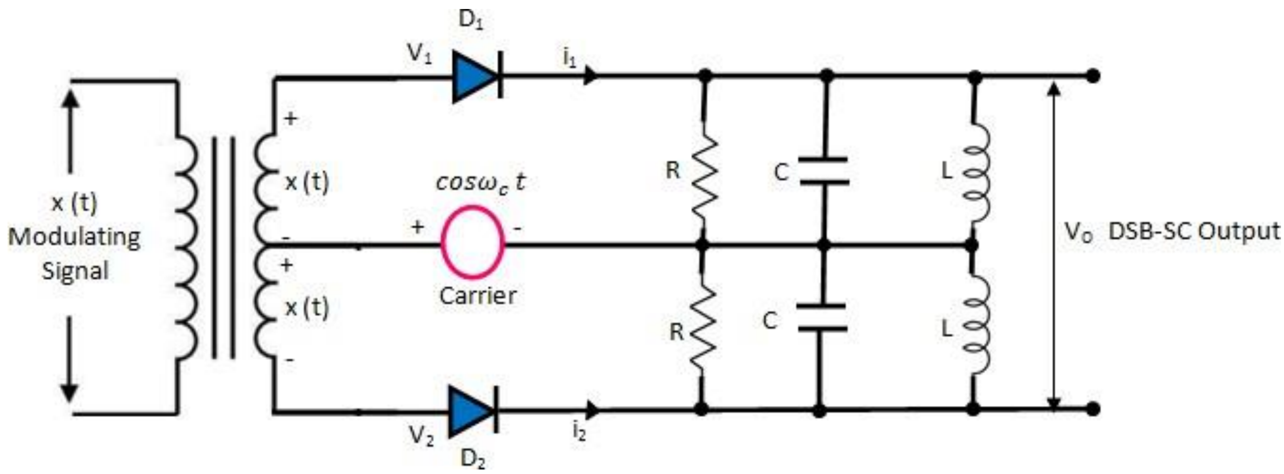


Fig 2.3: Balanced modulator

The modulating signal x(t) is applied equally with 180° phase reversal at the inputs of both the diodes through the input center tapped transformer .The carrier is applied to the center tap of the secondary .Hence, input voltage to D_1 is given by :

$$v_1 = cos\omega_c t + x(t)$$

And the input voltage to D_2 is given by :

$$v_2 = cos\omega_c t - x(t)$$

The diode current i_1 and i_2 are given by :

$$i_1 = av_1 + bv_1^2$$

$$i_1 = a[x(t) + cos\omega_c t] + b[x(t) + cos\omega_c t]^2$$

$$i_1 = ax(t) + acos\omega_c t + bx^2(t) + 2bx(t)cos\omega_c t + bcos^2\omega_c t$$

Similarly,

$$i_2 = av_2 + bv_2^2$$

$$i_2 = a[x(t) - cos\omega_c t] + b[x(t) - cos\omega_c t]^2$$

$$i_2 = av_2 + bv_2{}^2 = ax(t) - acos\omega_c t + bx^2(t) - 2bx(t)cos\omega_c t + bcos^2\omega_c t$$

The output voltage is given by :

$$v_o = i_1R - i_2R$$

substituting the expression for i1 and i2 from equations (3) and (4), we get

$$v_o = R[2\ a\ x(t) + 4\ b\ x(t)\ cos\omega_c\ t]$$

Or,

$$v_o = \underbrace{2aRx(t)}_{\text{Modulating Signal}} + \underbrace{4bRx(t)\ cos\omega_c t}_{\text{DSB-SC Signal}}$$

Hence, the output voltage contains a modulating signal term and the DSB-SC signal .The modulating signal term is eliminated and the second term is allowed to pass through to the output by the LC band pass filter section .

Therefore, final output = 4 b R x(t) cos ω_ct
= K x(t) cos ω_ct

Thus, the diode balanced modulator produces the DSB-SC signal at its output .

Ring Modulator

Ring modulator is the most widely used product modulator for generating DSBSC wave and is shown below.

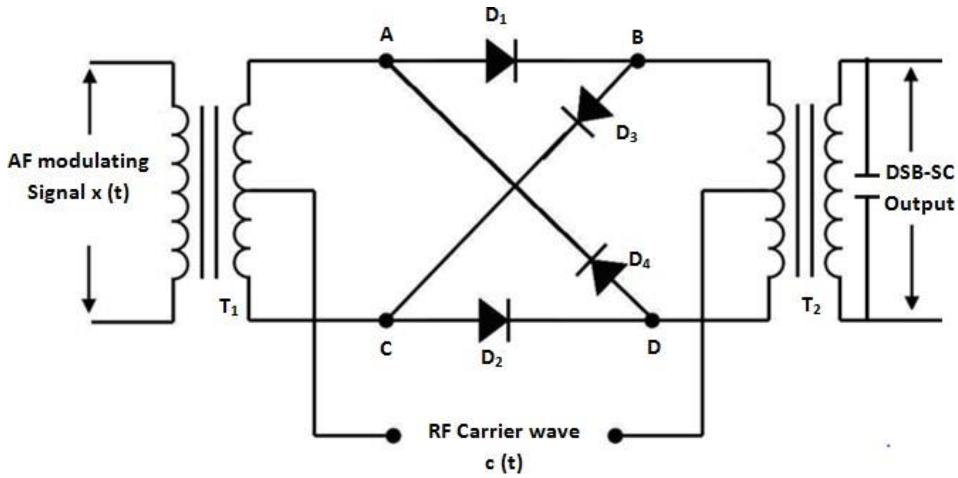


Fig 2.4: Ring Modulator

The four diodes form a ring in which they all point in the same direction. The diodes are controlled by square wave carrier c(t) of frequency fc, which is applied longitudinally by means of two center-tapped transformers. Assuming the diodes are ideal, when the carrier is positive, the outer diodes D1 and D2 are forward biased where as the inner diodes D3 and D4 are reverse biased, so that the modulator multiplies the base band signal m(t) by c(t). When the carrier is negative, the diodes D1 and D2 are reverse biased and D3 and D4 are forward, and the modulator multiplies the base band signal –m(t) by c(t).

Thus the ring modulator in its ideal form is a product modulator for square wave carrier and the base band signal m(t). The square wave carrier can be expanded using Fourier series as

$$c(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos(2\pi f_c t(2n-1))$$

Therefore the ring modulator out put is given by

$$s(t) = m(t)c(t)$$

$$s(t) = m(t)\left[\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos(2\pi f_c t(2n-1))\right]$$

From the above equation it is clear that output from the modulator consists entirely of modulation products. If the message signal m(t) is band limited to the frequency band $-w < f < w$, the output spectrum consists of side bands centred at fc.

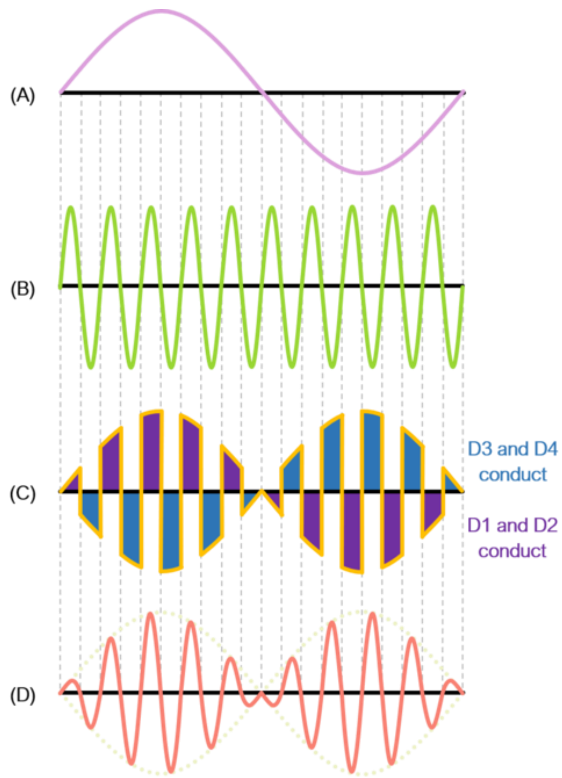


Fig 2.5: Wave forms Ring Modulator

Detection of DSB-SC waves:

Coherent Detection:

The message signal m(t) can be uniquely recovered from a DSBSC wave s(t) by first multiplying s(t) with a locally generated sinusoidal wave and then low pass filtering the product as shown.

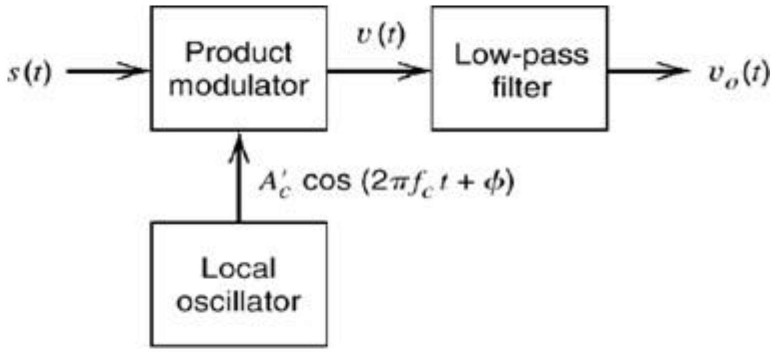


Fig 2.6: Coherent Detection of DSB-SC

It is assumed that the local oscillator signal is exactly coherent or synchronized, in both frequency and phase, with the carrier wave c(t) used in the product modulator to generate s(t). This method of demodulation is known as coherent detection or synchronous detection.

Let $A_c^1 \cos(2\pi f_c t + \phi)$ be the local oscillator signal, and $s(t) = A_c \cos(2\pi f_c t) m(t)$ be the DSBSC wave. Then the product modulator output $v(t)$ is given by

$$v(t) = A_c A_c^1 \cos(2\pi f_c t)\cos(2\pi f_c t + \phi) m(t)$$

$$v(t) = \frac{A_c A_c^1}{4}\cos(4\pi f_c t + \phi) m(t) + \frac{A_c A_c^1}{2}\cos(\phi) m(t)$$

The first term in the above expression represents a DSBSC modulated signal with a carrier frequency $2f_c$, and the second term represents the scaled version of message signal. Assuming that the message signal is band limited to the interval $-w < f < w$, the spectrum of $v(t)$ is plotted as shown below.

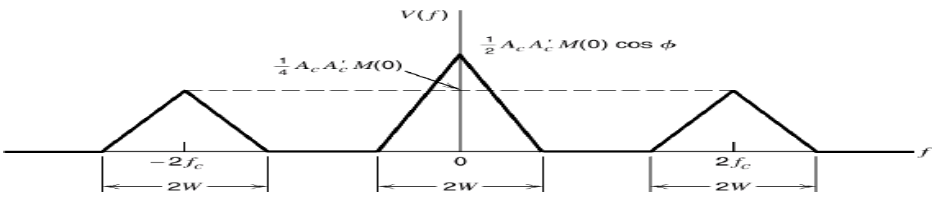


Fig.2.7.Spectrum of output of the product modulator

From the spectrum, it is clear that the unwanted component (first term in the expression) can be removed by the low-pass filter, provided that the cut-off frequency of the filter is greater than W but less than 2fc-W. The filter output is given by

$$v_o(t) = \frac{A_c A_c^{1}}{2} \cos(\phi) m(t)$$

The demodulated signal $v_o(t)$ is therefore proportional to m(t) when the phase error ϕ is constant.

Introduction of SSB-SC

Standard AM and DSBSC require transmission bandwidth equal to twice the message bandwidth. In both the cases spectrum contains two side bands of width W Hz, each. But the upper and lower sides are uniquely related to each other by the virtue of their symmetry about the carrier frequency. That is, given the amplitude and phase spectra of either side band, the other can be uniquely determined. Thus if only one side band is transmitted, and if both the carrier and the other side band are suppressed at the transmitter, no information is lost. This kind of modulation is called SSBSC and spectral comparison between DSBSC and SSBSC is shown in the figures.

Fig.2.8. Spectrum of DSB-SC

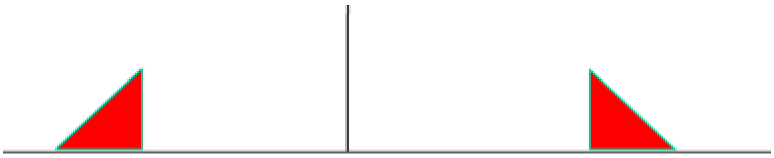


Fig.2.9. Spectrum of SSB-SC

Frequency Domain Description

Consider a message signal *m(t)* with a spectrum *M(f)* band limited to the interval $-w < f < w$ as shown in figure 3 , the DSBSC wave obtained by multiplexing *m(t)* by the carrier wave $c(t) = A_c \cos(2\pi f_c t)$and is also shown, in figure 4 . The upper side band is represented in duplicate by the frequencies above f_c and those below $-f_c$, and when only upper

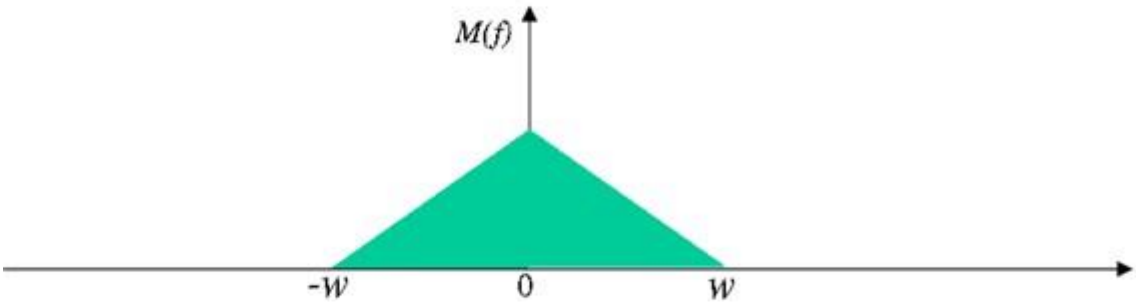


Fig.2.10. Message of SSB-SC

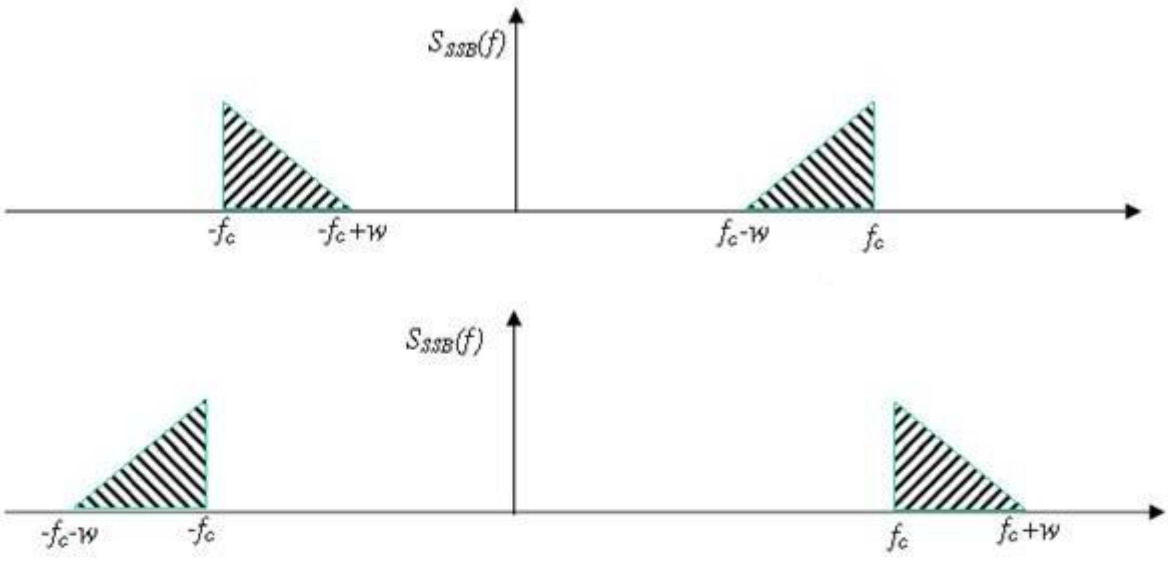


Fig.2.11. Spectrum of SSB-SC

side band is transmitted; the resulting SSB modulated wave has the spectrum shown in figure Similarly, the lower side band is represented in duplicate by the frequencies below fc and those above -fc and when only the lower side band is transmitted, the spectrum of the corresponding SSB modulated wave shown in figure2 . 1 1 .Thus the essential function of the SSB modulation is to translate the spectrum of the modulating wave, either with or without inversion, to a new location in the frequency domain. The advantage of SSB modulation is reduced bandwidth and the elimination of high power carrier wave. The main disadvantage is the cost and complexity of its implementation.

Hilbert Transform & its Properties:

The Fourier transform is useful for evaluating the frequency content of an energy signal, or in a limiting case that of a power signal. It provides mathematical basis for analyzing and designing the frequency selective filters for the separation of signals on the basis of their frequency content. In case of a sinusoidal signal, the simplest phase shift of 180^o is obtained by “Ideal transformer” (polarity reversal). When the phase angles of all the components of a given signal are shifted by 90^o, the resulting function of time is called the “Hilbert transform” of thesignal.

Consider an LTI system with transfer function defined by equation 1

$$H(f)=\begin{cases}-j, f>0\\ 0, f=0\\ j, f<0\end{cases} \qquad \text{-----------------} (1)$$

and the Signum function given by

$$\operatorname{sgn}(f)=\begin{cases}1, f>0\\ 0, f=0\\ -1, f<0\end{cases}$$

The function *H(f)* can be expressed using Signum function as given by 2

$$H(f)=-j\operatorname{sgn}(f) \qquad \text{---------------------} (2)$$

We know that $1e^{-j\pi/2} = -j$, $1e^{j\pi/2} = j$ and $e^{\pm j\theta} = \cos(\theta) \pm j\sin(\theta)$

Therefore,

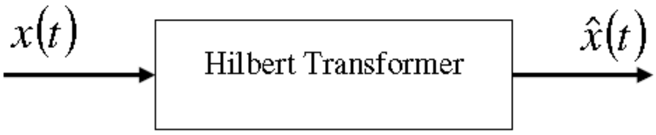
$$H(f) = \begin{cases} 1e^{-j\pi/2}, f > 0 \\ 1e^{j\pi/2}, f < 0 \end{cases}$$

Thus the magnitude $|H(f)| = 1,$ for all f, and angle

$$\angle H(f) = \begin{cases} -\pi/2, f > 0 \\ +\pi/2, f < 0 \end{cases}$$

The device which possesses such a property is called Hilbert transformer. Whenever a signal is applied to the Hilbert transformer, the amplitudes of all frequency components of the input signal remain unaffected. It produces a phase shift of -90° for all positive frequencies, while a phase shifts of 90° for all negative frequencies of the signal.

If x(t) is an input signal, then its Hilbert transformer is denoted by xˆ(t) and shown in the following diagram.



To find impulse response *h(t)* of Hilbert transformer with transfer function *H(f)*.

Consider the relation between Signum function and the unit step function.

$$\text{sgn}(t) = 2u(t) - 1 = x(t),$$

Differentiating both sides with respect to t,

$$\frac{d}{dt}\{x(t)\} = 2\delta(t)$$

Apply Fourier transform on both sides,

$$\text{sgn}(t) \leftrightarrow \frac{2}{j\omega} \longrightarrow \text{sgn}(t) \leftrightarrow \frac{1}{j\pi f}$$

Applying duality property of Fourier transform,

$$-Sgn(f) \leftrightarrow \frac{1}{j\pi t}$$

We have $$H(f) = -j\,\mathrm{sgn}(f)$$

$$H(f) \leftrightarrow \frac{1}{\pi t}$$

Therefore the impulse response *h(t)* of an Hilbert transformer is given by the equation 3 ,

$$h(t) = \frac{1}{\pi t} \qquad \text{------------------- (3)}$$

Now consider any input x(t) to the Hilbert transformer, which is an LTI system. Let the impulse response of the Hilbert transformer is obtained by convolving the input x(t) and impulse response h(t) of the system.

$$\hat{x}(t) = x(t) * h(t)$$

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{(t-\tau)} d\tau \qquad \text{------------------- (4)}$$

The equation 3.5 gives the Hilbert transform of *x(t)*.

The inverse Hilbert transform *x(t)* is given by

$$x(t) = \frac{-1}{\pi} \int_{-\infty}^{+\infty} \frac{\hat{x}(\tau)}{(t-\tau)} d\tau \qquad \text{------------------- (5)}$$

We have $$\hat{x}(t) = x(t) * h(t)$$

The Fourier transform $\hat{X}(f)$ of $\hat{x}(t)$ is given by

$$\hat{X}(f) = X(f)H(f)$$

$$\hat{X}(f) = -j\,\mathrm{sgn}(f)X(f) \qquad \text{------------------- (6)}$$

Properties:

1. "A signal *x(t)* and its Hilbert transform $\hat{x}(t)$ have the same amplitude spectrum".

The magnitude of *–jsgn(f)* is equal to 1 for all frequencies *f*. Therefore *x(t)* and $\hat{x}(t)$ have the same amplitude spectrum.

That is $$\left|\hat{X}(f)\right| = \left|X(f)\right| \qquad \text{for all } f$$

2. "If $\hat{x}(t)$ is the Hilbert transform of *x(t)*, then the Hilbert transform of $\hat{x}(t)$, is *–x(t)*".

To obtain its Hilbert transform of *x(t)*, *x(t)* is passed through a LTI system with a transfer function equal to *–jsgn(f)*. A double Hilbert transformation is equivalent to passing *x(t)* through a cascade of two such devices. The overall transfer function of such a cascade is equal to

$$\left[-j\operatorname{sgn}(f)\right]^2 = -1 \qquad \text{for all } f$$

The resulting output is *–x(t)*. That is the Hilbert transform of $\hat{x}(t)$ is equal to *–x(t)*.

Time Domain Description:

The time domain description of an SSB wave s(t) in the canonical form is given by the equation 1.

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \quad \text{-------------------- (1)}$$

where $S_I(t)$ is the in-phase component of the SSB wave and $S_Q(t)$ is its quadrature component. The in-phase component $S_I(t)$ except for a scaling factor, may be derived from *S(t)* by first multiplying *S(t)* by $\cos(2\pi f_c t)$ and then passing the product through a low-pass filter. Similarly, the quadrature component $S_Q(t)$, except for a scaling factor, may be derived from *s(t)* by first multiplying *s(t)* by $\sin(2\pi f_c t)$ and then passing the product through an identical filter.

The Fourier transformation of $S_I(t)$ and $S_Q(t)$ are related to that of SSB wave as follows, respectively.

$$S_I(f) = \begin{cases} S(f - f_c) + S(f + f_c), -w \le f \le w \\ 0, elsewhere \end{cases} \quad \text{------------------ (2)}$$

$$S_Q(f)=\begin{cases} j[S(f-f_c)-S(f+f_c)], -w \le f \le w \\ 0, elsewhere \end{cases}$$ ------------------- (3)

where $-w < f < w$ defines the frequency band occupied by the message signal $m(t)$.

Consider the SSB wave that is obtained by transmitting only the upper side band, shown in figure 10 . Two frequency shifted spectras $S(f-f_c)$ and $S(f+f_c)$ are shown in figure 11 and figure 12 respectively. Therefore, from equations 2 and 3 , it follows that the corresponding spectra of the in- phase component $S_I(t)$ and the quadrature component $S_Q(t)$ are as shown in figure 13 and 14 respectively.

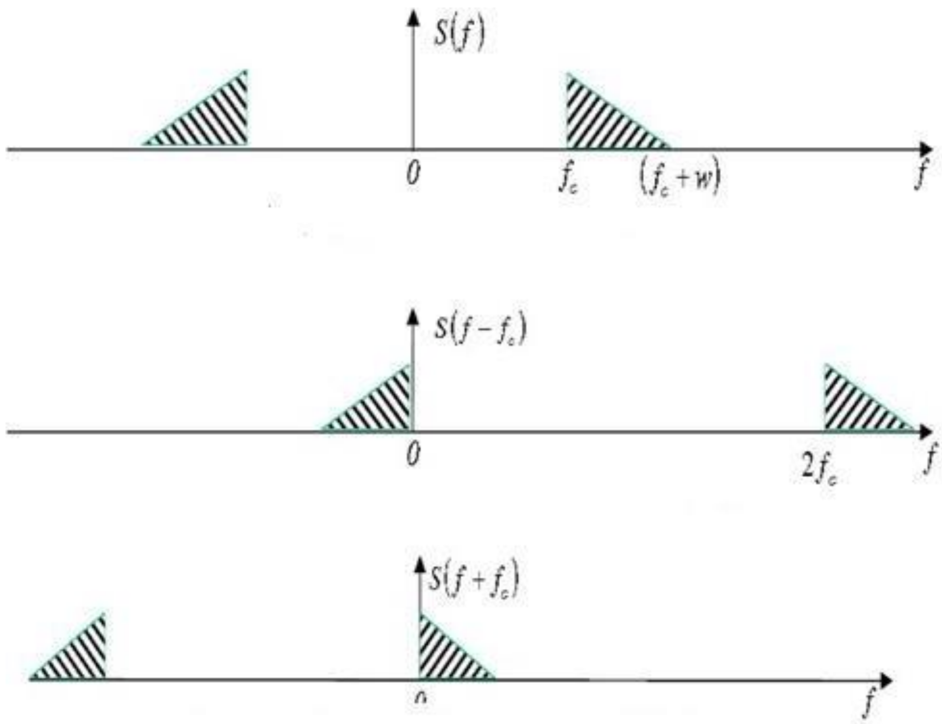


Fig.2.15. SSB-SC

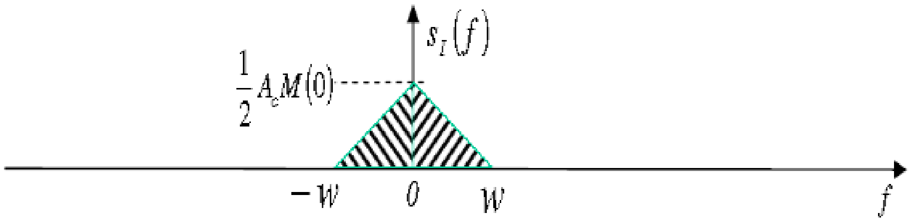


Figure 13 : Spectrum of in-phase component of SSBSC-USB

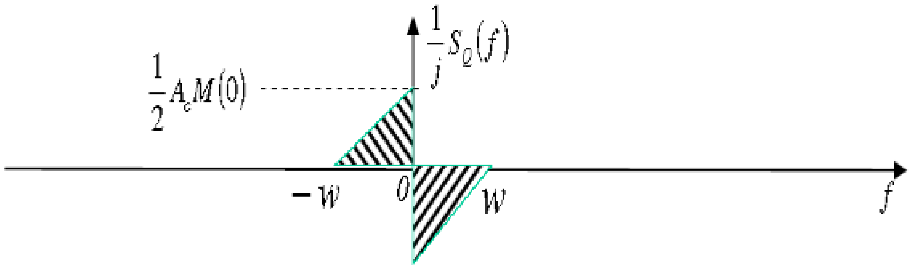


Figure 14 : Spectrum of quadrature component of SSBSC-USB

From the figure 13 , it is found that

$$S_I(f) = \frac{1}{2} A_c M(f)$$

where $M(f)$ is the Fourier transform of the message signal $m(t)$. Accordingly in-phase component $S_I(t)$ is defined by equation 4

$$\boxed{s_I(t) = \frac{1}{2} A_c m(t)} \qquad \text{------------------ (4)}$$

Now on the basis of figure14 , it is found that

$$S_Q(f) = \begin{cases} \frac{-j}{2} A_c M(f), f > 0 \\ 0, f = 0 \\ \frac{j}{2} A_c M(f), f < 0 \end{cases}$$

$$S_Q(f) = \frac{-j}{2} A_c \operatorname{sgn}(f) M(f) \qquad \text{------------------ (5)}$$

where $\operatorname{sgn}(f)$ is the Signum function.

But from the discussions on Hilbert transforms, it is shown that

$$-j\,\mathrm{sgn}(f)M(f) = \hat{M}(f) \qquad \text{------------------ (6)}$$

where $\hat{M}(f)$ is the Fourier transform of the Hilbert transform of $m(t)$. Hence the substituting equation (6) in (5), we get

$$S_Q(f) = \frac{1}{2}A_c\hat{M}(f) \qquad \text{------------------------ (7)}$$

Therefore quadrature component $s_Q(t)$ is defined by equation 8

$$s_Q(t) = \frac{1}{2}A_c\hat{m}(t) \qquad \text{------------------------- (8)}$$

Therefore substituting equations (4) and (8) in equation in (1), we find that canonical representation of an SSB wave $s(t)$ obtained by transmitting only the upper side band is given by the equation 9

$$s_U(t) = \frac{1}{2}A_c m(t)\cos(2\pi f_c t) - \frac{1}{2}A_c\hat{m}(t)\sin(2\pi f_c t) \qquad \text{------------- (9)}$$

Following the same procedure, we can find the canonical representation for an SSB wave
s(t) obtained by transmitting only the lower side band is given by

$$s_L(t) = \frac{1}{2}A_c m(t)\cos(2\pi f_c t) + \frac{1}{2}A_c\hat{m}(t)\sin(2\pi f_c t) \qquad \text{-------------- (10)}$$

Generation of SSB wave:

Frequency discrimination method

Consider the generation of SSB modulated signal containing the upper side band only. From a practical point of view, the most severe requirement of SSB generation arises from the unwanted sideband, the nearest component of which is separated from the desired side band by twice the lowest frequency component of the message signal. It implies that, for the generation of an SSB wave to be possible, the message spectrum must have an energy gap centered at the origin as shown in figure. This requirement is naturally satisfied by voice signals, whose energy gap is about 600Hz wide

The frequency discrimination or filter method of SSB generation consists of a product modulator, which produces DSBSC signal and a band-pass filter to extract the desired side band and reject the other and is shown in the figure 8.

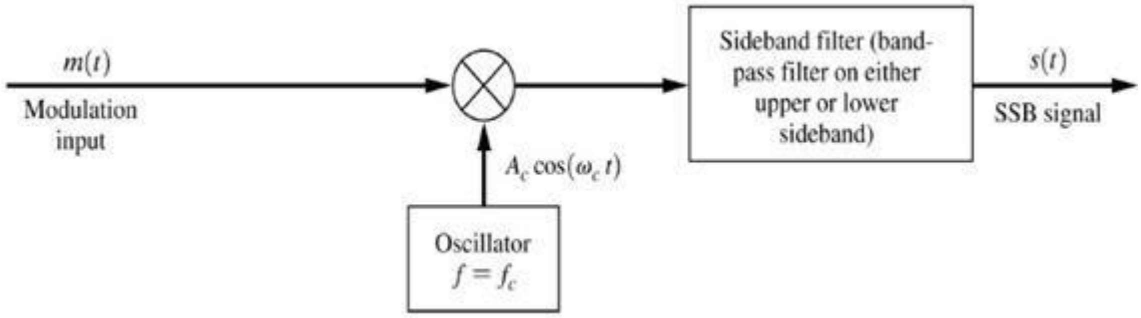


Fig.2.14. Frequency Discrimination Method of SSB-SC

Application of this method requires that the message signal satisfies two conditions:
1. The message signal m(t) has no low-frequency content. Example: speech, audio, music.
2. The highest frequency component W of the message signal m(t) is much less than the carrier frequency fc.

Then, under these conditions, the desired side band will appear in a non-overlapping interval in the spectrum in such a way that it may be selected by an appropriate filter.

In designing the band pass filter, the following requirements should be satisfied:

1.The pass band of the filter occupies the same frequency range as the spectrum of the desired SSB modulated wave.

2. The width of the guard band of the filter, separating the pass band from the stop band, where the unwanted sideband of the filter input lies, is twice the lowest frequency component of the message signal.

When it is necessary to generate an SSB modulated wave occupying a frequency band that is much higher than that of the message signal, it becomes very difficult to design an appropriate filter that will pass the desired side band and reject the other. In such a situation it is necessary to resort to a multiple-modulation process so as to ease the filtering requirement. This approach is illustrated in the following figure 2.15 involving two stages of modulation.

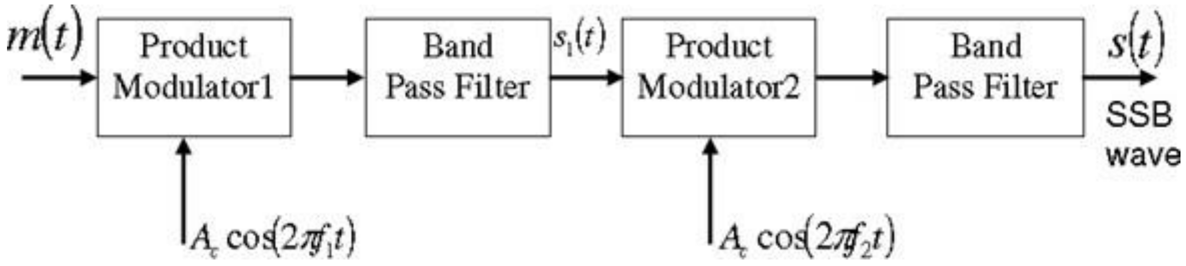


Fig.2.15. Two Stage Frequency Discrimination Method of SSB-SC

The SSB modulated wave at the first filter output is used as the modulating wave for the second product modulator, which produces a DSBSC modulated wave with a spectrum that is symmetrically spaced about the second carrier frequency f2. The frequency separation between the side bands of this DSBSC modulated wave is effectively twice the first carrier frequency f1, thereby permitting the second filter to remove the unwanted side band.

Phase discrimination method for generating SSB wave:

Time domain description of SSB modulation leads to another method of SSB generation using the equations 9 or 10. The block diagram of phase discriminator is as shown in figure2.16.

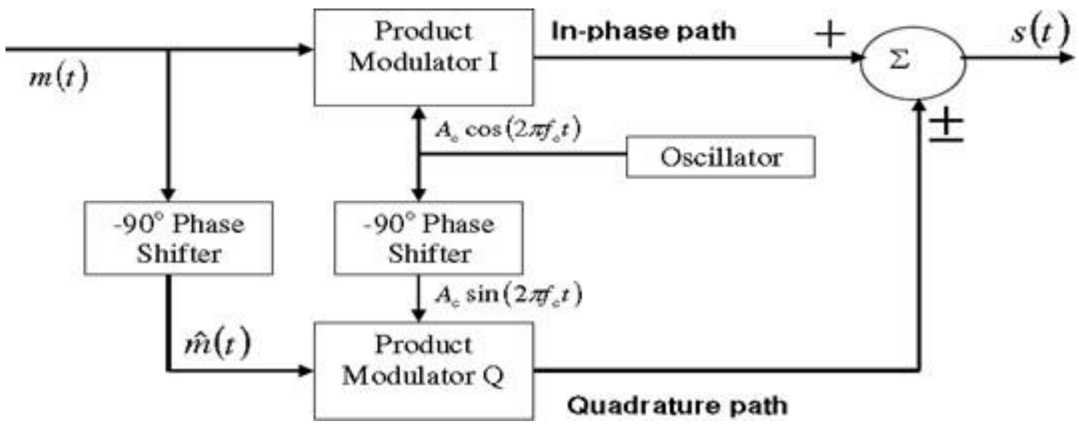


Fig.2.16. Block Diagram Of Phase Discriminator

The phase discriminator consists of two product modulators I and Q, supplied with carrier waves in-phase quadrature to each other. The incoming base band signal m(t) is applied to product modulator I, producing a DSBSC modulated wave that contains reference phase sidebands symmetrically spaced about carrier frequency fc.

The Hilbert transform mˆ (t) of m (t) is applied to product modulator Q, producing a DSBSC modulated that contains side bands having identical amplitude spectra to those of modulator I, but with phase spectra such that vector addition or subtraction of the two modulator outputs results in cancellation of one set of side bands and reinforcement of the other set.

The use of a plus sign at the summing junction yields an SSB wave with only the lower side band, whereas the use of a minus sign yields an SSB wave with only the upper side band. This modulator circuit is called Hartley modulator.

Demodulation of SSB Waves:

Demodulation of SSBSC wave using coherent detection is as shown in . The SSB wave *s(t)* together with a locally generated carrier $c(t) = A_c^1 \cos(2\pi f_c t + \phi)$ is applied to a product modulator and then low-pass filtering of the modulator output yields the message signal.

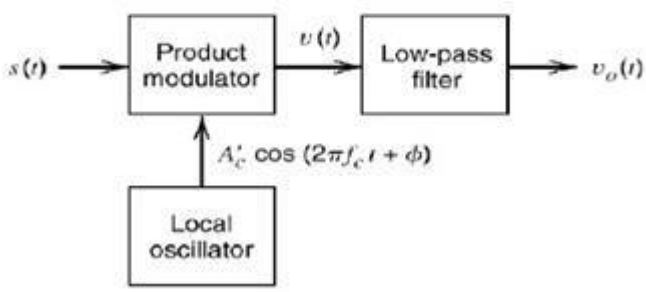


Fig.2.17. Block Diagram Of Coherent Detection Of Ssb-Sc

$$v(t) = A_c^1 \cos(2\pi f_c t + \phi) s(t) \,, \quad \text{Put } \phi = 0$$

$$v(t) = \frac{1}{2} A_c \cos(2\pi f_c t)[m(t)\cos(2\pi f_c t) \pm \hat{m}(t)\sin(2\pi f_c t)]$$

$$v(t) = \frac{1}{4} A_c m(t) + \frac{1}{4} A_c [m(t)\cos(4\pi f_c t) \pm \hat{m}(t)\sin(4\pi f_c t)] \quad \text{..............(1)}$$

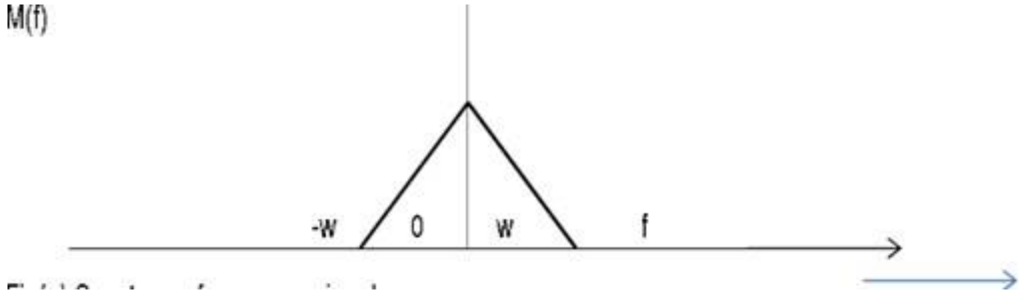
The first term in the above equation 1 is desired message signal. The other term represents an SSB wave with a carrier frequency of $2f_c$ as such; it is an unwanted component, which is removed by low-pass filter.

Introduction to Vestigial Side Band Modulation

Vestigial sideband is a type of Amplitude modulation in which one side band is completely passed along with trace or tail or vestige of the other side band. VSB is a compromise between SSB and DSBSC modulation. In SSB, we send only one side band, the Bandwidth required to send SSB wave is w. SSB is not appropriate way of modulation when the message signal contains significant components at extremely low frequencies. To overcome this VSB is used.

Frequency Domain Description;

The following Fig illustrates the spectrum of VSB modulated wave s (t) with respect to the message m (t) (band limited)



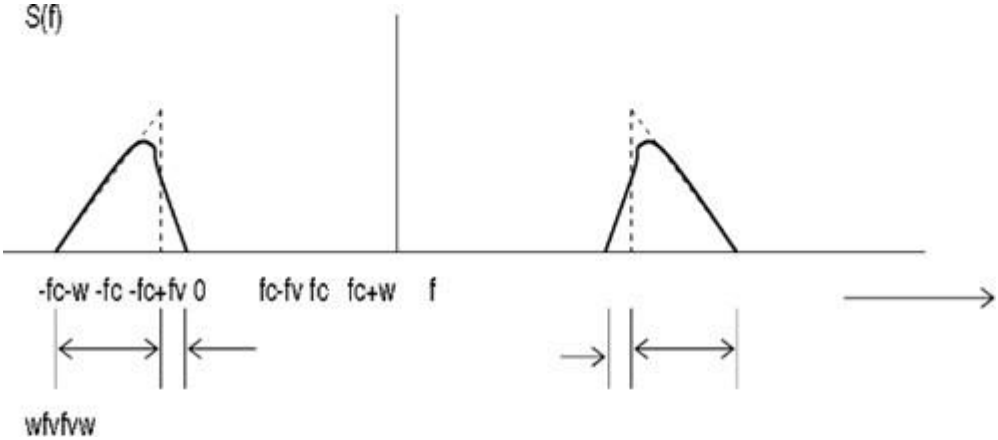


Fig2.18: spectrum of vsb contaning lower side band

Assume that the Lower side band is modified into the vestigial side band. The vestige of the lower sideband compensates for the amount removed from the upper sideband. The bandwidth required to send VSB wave is

$B = w + f_v$

Where f_v is the width of the vestigial side band.

Similarly, if Upper side band is modified into the vestigial side band then,

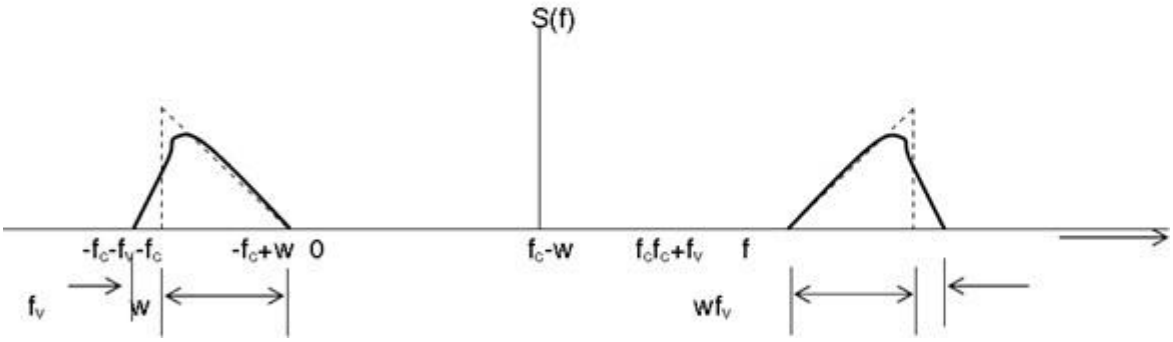


Fig2.19: spectrum of vsb contaning upper side band

The vestige of the Upper sideband compensates for the amount removed from the Lower sideband. The bandwidth required to send VSB wave is B = w+fv, where fv is the width of the vestigial side band.

Therefore, VSB has the virtue of conserving bandwidth almost as efficiently as SSB modulation, while retaining the excellent low-frequency base band characteristics of DSBSC and it is standard for the transmission of TV signals.

Generation of VSB Modulated Wave

VSB modulated wave is obtained by passing DSBSC through a sideband shaping filter as shown in fig below.

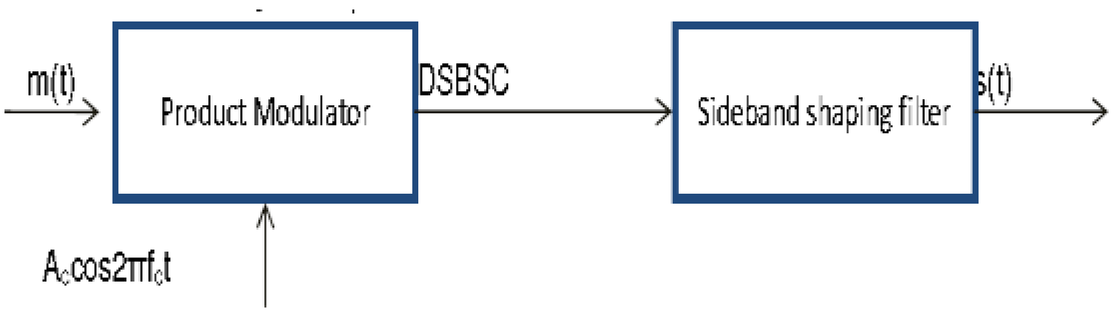


Fig2.20. Block Diagram of VSB Modulator

The exact design of this filter depends on the spectrum of the VSB waves. The relation between filter transfer function H (f) and the spectrum of VSB waves is given by

$$S(f) = Ac/2\,[M(f - fc) + M(f + fc)]H(f) \text{-------------------------} (1)$$

Where M(f) is the spectrum of Message Signal. Now, we have to determine the specification for the filter transfer function H(f) It can be obtained by passing s(t) to a coherent detector and determining the necessary condition for undistorted version of the message signal m(t). Thus, s (t) is multiplied by a locally generated sinusoidal wave cos $(2\pi f_c t)$ which is synchronous with the carrier wave $A_c\cos(2\pi f_c t)$ in both frequency and phase, as in fig below,

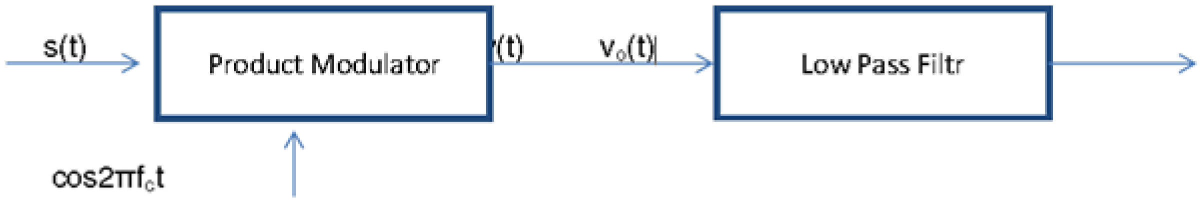


Fig2.21. Block Diagram of VSB Demodulator

Then, $v(t) = s(t).\cos 2\pi f_c t$-----------------------------------(2)

In frequency domain Eqn (2) becomes,

$V(f) = ½ [S(f - f_c) + S(f + f_c)]$ --(3)

Substitution of Eqn (1) in Eqn (3) gives

$$V(f) = ½[A_c /2 [M (f - f_c - f_c) + M(f - f_c + f_c)]H(f – f_c)$$
$$+ ½[A_c /2 [M (f + f_c - f_c) + M(f + f_c + f_c)]H(f + f_c)$$

$$V(f) = ½[A_c /2 [M (f -2 f_c) + M(f)]H(f – f_c)$$
$$+ ½[A_c /2 [M (f) + M(f +2f_c)]H(f + f_c)$$

$$V(f) = A_c /4\, M(f)[H (f - f_c) + H(f + f_c)]$$
$$+ A_c /4\, [M(f -2 f_c) H (f - f_c) + M(f + 2f_c) H(f + f_c)]$$ -------------------------------(4)

The spectrum of V(f) as shown in fig below,

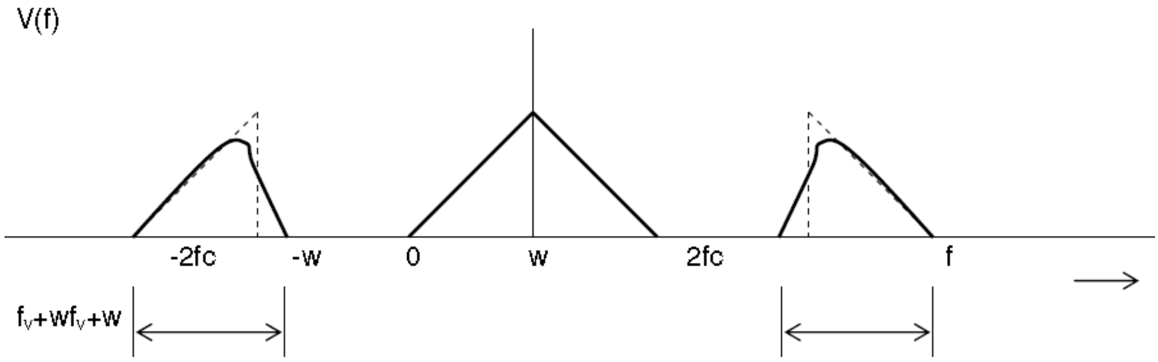


Fig ©. Spectrum of the product modulator output v(t)

Pass v(t) to a Low pass filter to eliminate VSB wave corresponding to $2f_c$.

$V_o(f) = A_c /4\, M(f)[H (f - f_c) + H(f + f_c)]$ ---(5)

The spectrum of Vo (f) is in fig below,

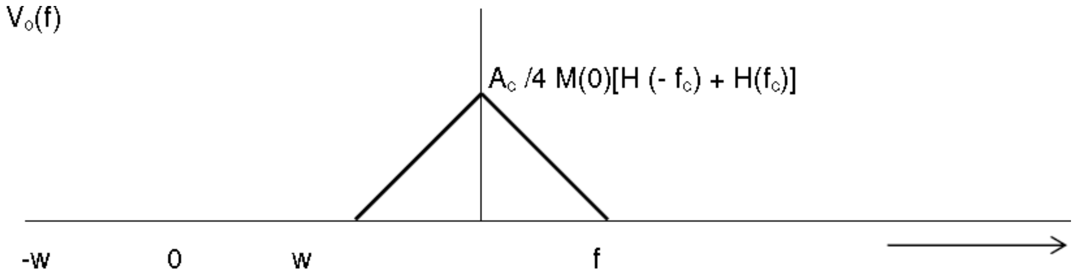


Fig (d). Spectrum of the demodulated Signal $v_o(t)$.

For a distortion less reproduction of the original signal m(t), $V_o(f)$ to be a scaled version of M(f). Therefore, the transfer function H(f) must satisfy the condition

$$H(f - f_c) + H(f + f_c) = 2H(f_c) \text{------------------} (6)$$

Where $H(f_c)$ is a constant

Since m(t) is a band limited signal, we need to satisfy eqn (6) in the interval -w≤f≤w. The requirement of eqn (6) is satisfied by using a filter whose transfer function is shown below

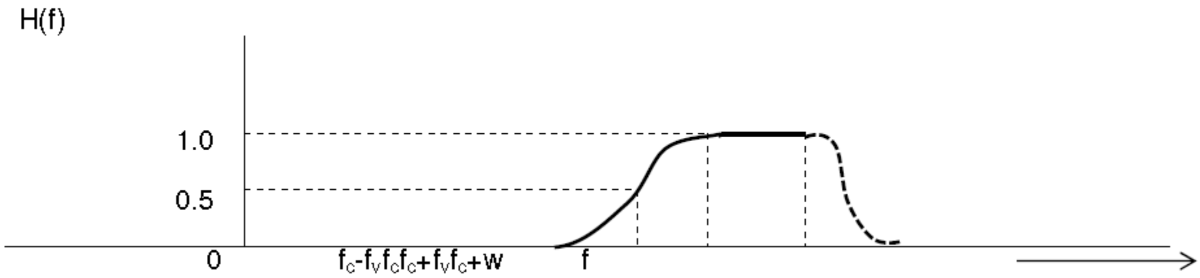


Fig (e) Frequency response of sideband shaping filter

Note: H(f) is Shown for positive frequencies only.

The Response is normalized so that H(f) at f_c is 0.5. Inside this interval $f_c - f_v \le f \le f_c + f_v$ response exhibits odd symmetry. i.e., Sum of the values of H(f) at any two frequencies equally displaced above and below is Unity.

Similarly, the transfer function H (f) of the filter for sending Lower sideband along with the vestige of the Upper sideband is shown in fig below,

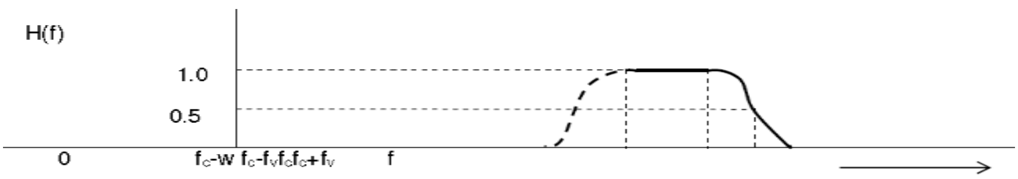


Fig (f) Frequency response of sideband shaping filter

Note: H(f) is Shown for positive frequencies only.

Comparison of AM Techniques:

Parameter of Comparison	DSBFC	DSBSC	SSB	VSB
Carrier Suppression	NA	Fully	Fully	NA
Sideband Suppression	NA	NA	One SB completely	One SB suppressed partially
Bandwidth	$2f_m$	$2f_m$	f_m	f_m < BW > $2f_m$
Transmission efficiency	Minimum	Moderate	Maximum	Moderate
Number of modulating inputs	1	1	1	2
Applications	Radio broadcasting	Radio broadcasting	Point to point mobile communication	TV

Applications of different AM systems:

- Amplitude Modulation: AM radio, Short wave radio broadcast
- DSB-SC: Data Modems, Color TV's color signals.
- SSB: Telephone
- VSB: TV picture signals

UNIT III
ANGLE MODULATION

- Generalized concept of angle modulation
- Frequency Modulation
- Single tone frequency modulation
- Narrow band FM
- Wide band FM
- Generation of FM Waves:
    - Indirect FM,
    - Direct FM: Varactor Diode and ReactanceModulator
- Detection of FM Waves:
    - Balanced Frequency discriminator,
    - Zero crossing detector,
    - Phase locked loop
- Pre-emphasis & de-emphasis
- FM Transmitter block diagram and explanation of each block

Generalized concept of angle modulation

Instantaneous Frequency

The frequency of a cosine function $x(t)$ that is given by

$$x(t) = \cos(\omega_c t + \theta_0)$$

is equal to ω_c since it is a constant with respect to t, and the phase of the cosine is the constant θ_0. The angle of the cosine $\theta(t) = \omega_c t + \theta_0$ is a linear relationship with respect to t (a straight line with slope of ω_c and y–intercept of θ_0). However, for other sinusoidal functions, the frequency may itself be a function of time, and therefore, we should not think in terms of the constant frequency of the sinusoid but in terms of the INSTANTANEOUS frequency of the sinusoid since it is not constant for all t. Consider for example the following sinusoid

$$y(t) = \cos[\theta(t)],$$

where $\theta(t)$ is a function of time. The frequency of $y(t)$ in this case depends on the function of $\theta(t)$ and may itself be a function of time. The instantaneous frequency of $y(t)$ given above is defined as

$$\omega_i(t) = \frac{d\theta(t)}{dt}.$$

As a checkup for this definition, we know that the instantaneous frequency of $x(t)$ is equal to its frequency at all times (since the instantaneous frequency for that function is constant) and is equal to ω_c. Clearly this satisfies the definition of the instantaneous frequency since $\theta(t) = \omega_c t + \theta_0$ and therefore $\omega_i(t) = \omega_c$.

If we know the instantaneous frequency of some sinusoid from $-\infty$ to sometime t, we can find the angle of that sinusoid at time t using

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha) d\alpha.$$

Changing the angle $\theta(t)$ of some sinusoid is the bases for the two types of angle modulation: Phase and Frequency modulation techniques.

Phase Modulation (PM)

In this type of modulation, the phase of the carrier signal is directly changed by the message signal. The phase modulated signal will have the form

$$g_{PM}(t) = A \cdot \cos\left[\omega_c t + k_p m(t)\right],$$

where A is a constant, ω_c is the carrier frequency, $m(t)$ is the message signal, and k_p is a parameter that specifies how much change in the angle occurs for every unit of change of $m(t)$. The phase and instantaneous frequency of this signal are

$$\theta_{PM}(t) = \omega_c t + k_p m(t),$$
$$\omega_i(t) = \omega_c + k_p \frac{dm(t)}{dt} = \omega_c + k_p \dot{m}(t).$$

So, the frequency of a PM signal is proportional to the derivative of the message signal.

Frequency Modulation (FM)

This type of modulation changes the frequency of the carrier (not the phase as in PM) directly with the message signal. The FM modulated signal is

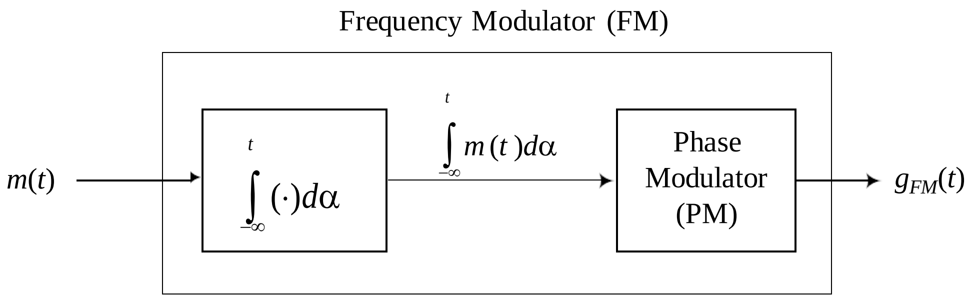
$$g_{FM}(t) = A \cdot \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right],$$

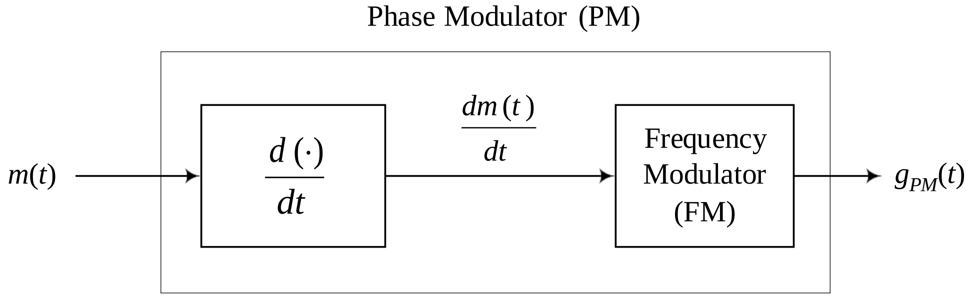
where k_f is a parameter that specifies how much change in the frequency occurs for every unit change of $m(t)$. The phase and instantaneous frequency of this FM are

$$\theta_{FM}(t) = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha,$$
$$\omega_i(t) = \omega_c + k_f \frac{d}{dt}\left[\int_{-\infty}^{t} m(\alpha)\,d\alpha\right] = \omega_c + k_f m(t).$$

Relation between PM and FM

PM and FM are tightly related to each other. We see from the phase and frequency relations for PM and FM given above that replacing $m(t)$ in the PM signal with $\int_{-\infty}^{t} m(\alpha)\,d\alpha$ gives an FM signal and replacing $m(t)$ in the FM signal with $\frac{dm(t)}{dt}$ gives a PM signal. This is illustrated in the following block diagrams.





Frequency Modulation

In **F**requency **M**odulation (**FM**) the instantaneous value of the information signal controls the frequency of the carrier wave. This is illustrated in the following diagrams.

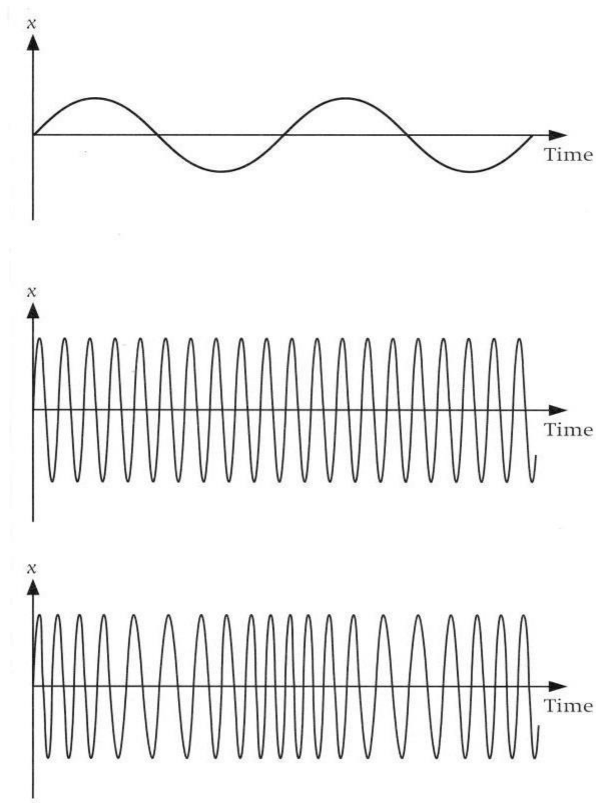


Fig 3.1 wave forms of frequency moduation

Notice that as the information signal increases, the frequency of the carrier increases, and as the information signal decreases, the frequency of the carrier decreases.

The frequency f_i of the information signal controls the rate at which the carrier frequency increases and decreases. As with AM, f_i must be less than f_c. The amplitude of the carrier remains constant throughout this process.

When the information voltage reaches its maximum value then the change in frequency of the carrier will have also reached its maximum deviation above the nominal value. Similarly when the information reaches a minimum the carrier will be at its lowest frequency below the nominal carrier frequency value. When the information signal is zero, then no deviation of the carrier will occur.

The maximum change that can occur to the carrier from its base value f_c is called the frequency deviation, and is given the symbol Δf_c. This sets the dynamic range (i.e. voltage range) of the transmission. The dynamic range is the ratio of the largest and smallest analogue information signals that can be transmitted.

Bandwidth of FM and PM Signals

The bandwidth of the different AM modulation techniques ranges from the bandwidth of the message signal (for SSB) to twice the bandwidth of the message signal (for DSBSC and Full AM). When FM signals were first proposed, it was thought that their bandwidth can be reduced to an arbitrarily small value. Compared to the bandwidth of different AM modulation techniques, this would in theory be a big advantage. It was assumed that a signal with an instantaneous frequency that changes over of range of Δf Hz would have a bandwidth of Δf Hz. When experiments were done, it was discovered that this was not the case. It was discovered that the bandwidth of FM signals for a specific message signal was at least equal to the bandwidth of the corresponding AM signal. In fact, FM signals can be classified into two types: Narrowband and Wideband FM signals depending on the bandwidth of each of these signals

Narrowband FM and PM

The general form of an FM signal that results when modulating a signals $m(t)$ is

$$g_{FM}(t) = A \cdot \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right].$$

A narrow band FM or PM signal satisfies the condition

$$|k_f a(t)| \ll 1$$

For FM and

$$\left|k_p \cdot m(t)\right| \ll 1$$

For PM, where

$$a(t) = \int_{-\infty}^{t} m(\alpha)d\alpha ,$$

such that a change in the message signal does not results in a lot of change in the instantaneous frequency of the FM signal.

Now, we can write the above as

$$g_{FM}(t) = A \cdot \cos\left[\omega_c t + k_f a(t)\right].$$

Starting with FM, to evaluate the bandwidth of this signal, we need to expand it using a power series expansion. So, we will define a slightly different signal

$$\hat{g}_{FM}(t) = A \cdot e^{j\{\omega_c t + k_f a(t)\}} = A \cdot e^{j\omega_c t} \cdot e^{jk_f a(t)}.$$

Remember that

$$\hat{g}_{FM}(t) = A \cdot e^{j\{\omega_c t + k_f a(t)\}} = A \cdot \cos\left[\omega_c t + k_f a(t)\right] + jA \cdot \sin\left[\omega_c t + k_f a(t)\right],$$

so

$$g_{FM}(t) = \text{Re}\left\{\hat{g}_{FM}(t)\right\}.$$

Now we can expand the term $e^{jk_f a(t)}$ in $\hat{g}_{FM}(t)$, which gives

$$\hat{g}_{FM}(t) = A \cdot e^{j\omega_c t} \cdot \left[1 + jk_f a(t) + \frac{j^2 k_f^2 a^2(t)}{2!} + \frac{j^3 k_f^3 a^3(t)}{3!} + \frac{j^4 k_f^4 a^4(t)}{4!} + \dots\right]$$
$$= A \cdot \left[e^{j\omega_c t} + jk_f a(t) e^{j\omega_c t} - \frac{k_f^2 a^2(t)}{2!} e^{j\omega_c t} - \frac{jk_f^3 a^3(t)}{3!} e^{j\omega_c t} + \frac{k_f^4 a^4(t)}{4!} e^{j\omega_c t} + \dots\right]$$

Since k_f and $a(t)$ are real ($a(t)$ is real because it is the integral of a real function $m(t)$), and since $\text{Re}\{e^{j\omega_c t}\} = \cos(\omega_c t)$ and $\text{Re}\{je^{j\omega_c t}\} = -\sin(\omega_c t)$, then

$$g_{FM}(t) = \mathrm{Re}\{\hat{g}_{FM}(t)\}$$
$$= A\cdot\left[\cos(\omega_c t) - k_f a(t)\sin(\omega_c t) - \frac{k_f^2 a^2(t)}{2!}\cos(\omega_c t) + \frac{k_f^3 a^3(t)}{3!}\sin(\omega_c t) + \frac{k_f^4 a^4(t)}{4!}\cos(\omega_c t) + \ldots\right]$$

The assumption we made for narrowband FM is ($|k_f a(t)| \ll 1$). This assumption will result in making all the terms with powers of $k_f a(t)$ Greater than 1 to be small compared to the first two terms. So, the following is a reasonable approximation for $g_{FM}(t)$

$$\boxed{g_{FM\ (Narrowband)}(t) \approx A\cdot\left[\cos(\omega_c t) - k_f a(t)\sin(\omega_c t)\right],} \qquad \text{when } |k_f a(t)| \ll 1.$$

It must be stressed that the above approximation is only valid for narrowband FM signals that satisfy the condition ($|k_f a(t)| \ll 1$). The above signal is simply the addition (or actually the subtraction) of a cosine (the carrier) with a DSBSC signal (but using a sine as the carrier). The message signal that modulates the DSBSC signal is not $m(t)$ but its integration $a(t)$. One of the properties of the Fourier transform informs us that the bandwidth of a signal $m(t)$ and its integration $a(t)$ (and its derivative too) are the same (verify this). Therefore, the bandwidth of the narrowband FM signal is

$$\boxed{BW_{FM\ (Narrowband)} = BW_{DSBSC} = 2\cdot BW_{m(t)} = }.$$

We will see later that when the condition ($k_f \ll 1$) is not satisfied, the bandwidth of the FM signal becomes higher that twice the bandwidth of the message signal. Similar relationships hold for PM signals. That is

$$\boxed{g_{PM\ (Narrowband)}(t) \approx A\cdot\left[\cos(\omega_c t) - k_p m(t)\sin(\omega_c t)\right],} \qquad \text{when } |k_p \cdot m(t)| \ll 1,$$

and

$$\boxed{BW_{PM\ (Narrowband)} = BW_{DSBSC} = 2\cdot BW_{m(t)} = }.$$

<u>Construction of Narrowband Frequency and Phase Modulators</u>

The above approximations for narrowband FM and PM can be easily used to construct modulators for both types of signals

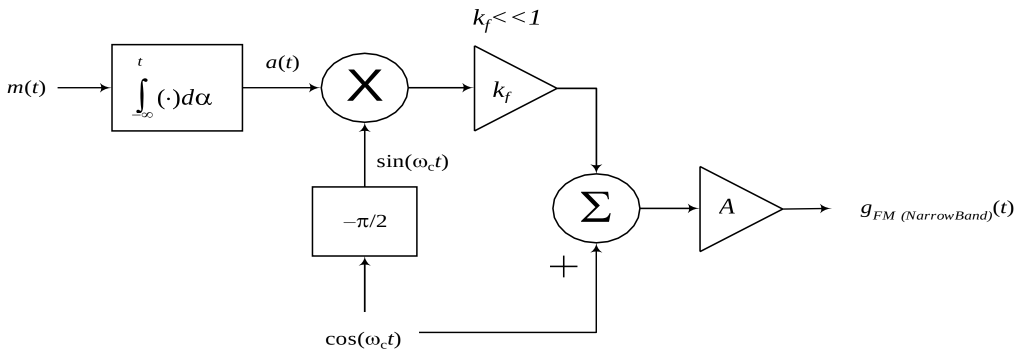


fig 3.2:Narrowband FM Modulator

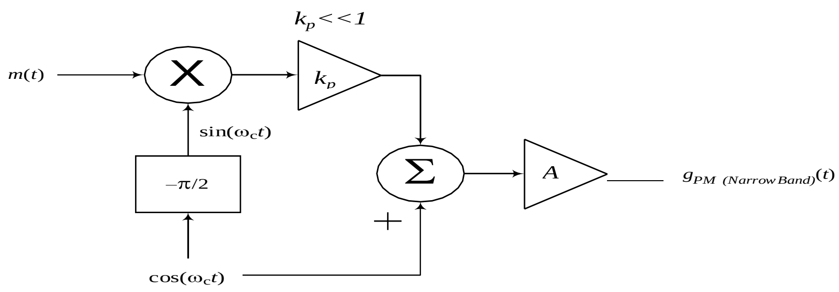


fig 3.3:Narrowband PM Modulator

Generation of Wideband FM Signals

Consider the following block diagram

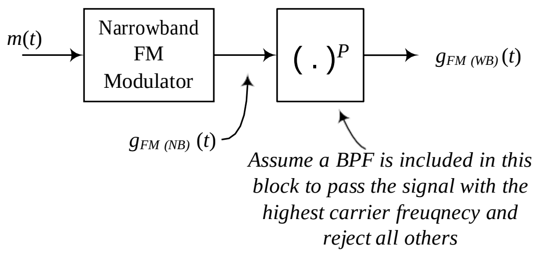


fig 3.4:wide band FM Modulator

A narrowband FM signal can be generated easily using the block diagram of the narrowband FM modulator that was described in a previous lecture. The narrowband FM modulator generates a narrowband FM signal using simple components such as an integrator (an OpAmp), oscillators, multipliers, and adders. The generated narrowband FM signal can be converted to a wideband FM signal by simply passing it through a non–linear device with power P. Both the carrier frequency and the frequency deviation Δf of the narrowband signal are increased by a factor *P*. Sometimes, the desired increase in the carrier frequency and the desired increase in Δf are different. In this case, we increase Δf to the desired value and use a frequency shifter (multiplication by a sinusoid followed by a BPF) to change the carrier frequency to the desired value.

SINGLE-TONE FREQUENCY MODULATION

Time-Domain Expression

Since the FM wave is a nonlinear function of the modulating wave, the frequency modulation is a nonlinear process. The analysis of nonlinear process is the difficult task. In this section, we will study single-tone frequency modulation in detail to simplify the analysis and to get thorough understanding about FM.

Let us consider a single-tone sinusoidal message signal defined by

$$n(t) = A_n \cos(2nf_nt) \qquad (5.13)$$

The instantaneous frequency from Eq. (5.8) is then

$$f(t) = f_c + k_f A_n \cos(2nf_nt) = f_c + \Delta f \cos(2nf_nt) \qquad (5.14)$$

where

$$\Delta f = k_f A_n$$

$$\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m t) dt$$

$$= 2\pi f_c t + 2\pi k_f \frac{A_m}{2\pi f_m} \sin(2\pi f_m t)$$

$$= 2\pi f_c t + \frac{k_f A_m}{f_m} \sin(2\pi f_m t)$$

$$= 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)$$

$$\therefore \theta(t) = 2\pi f_c t + \beta_f \sin(2\pi f_m t)$$

Where

$$\beta_f = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

is the modulation index of the FM wave. Therefore, the single-tone FM wave is expressed by

$$s_{FM}(t) = A_c \cos[2nf_c t + \text{þ}_f \sin(2nf_n t)] \qquad (5.18)$$

This is the desired time-domain expression of the single-tone FM wave

Similarly, **single-tone phase modulated wave** may be determined from Eq.as

$$s_{PM}(t) = A_c \cos[2nf_c t + k_p A_n \cos(2nf_n t)]$$

$$\text{or, } s_{PM}(t) = A_c \cos[2nf_c t + \text{þ}_p \cos(2nf_n t)] \qquad (5.19)$$

where

$$\text{þ}_p = k_p A_n \qquad (5.20)$$

is the modulation index of the single-tone phase modulated wave.

The frequency deviation of the single-tone PM wave is

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta_f \sin(2\pi f_m t)\right]$$

Spectral Analysis of Single-Tone FM Wave

The above Eq. can be rewritten as

$$s_{FM}(t) = \mathrm{Re}\{A_c e^{j2\pi f_c t} e^{j\beta \sin(2\pi f_m t)}\}$$

For simplicity, the modulation index of FM has been considered as β instead of β_f afterward. Since $\sin(2\pi f_m t)$ is periodic with fundamental period $T = 1/f_m$, the complex expontial $e^{j\beta \sin(2\pi f_m t)}$ is also periodic with the same fundamental period. Therefore, this complex exponential can be expanded in Fourier series representation as

$$e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t}$$

where the Fourier series coefficients c_n are obtained as

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} e^{j\beta\sin(2\pi f_m t)} e^{-j2\pi n f_m t} dt \quad (5.24)$$

Let $2\pi f_m t = x$, then Eq. (5.24) reduces to

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\beta\sin(x)} e^{-jnx} dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin(x)-nx)} dx \quad (5.25)$$

The integral on the right-hand side is known as the nth order Bessel function of the first kind and is denoted by $J_n(\beta)$. Therefore, $c_n = J_n(\beta)$ and Eq. (4.23) can be written as

$$e^{j\beta\sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} \quad (5.26)$$

By substituting Eq. (5.26) in Eq. (5.22), we get

$$\begin{aligned} s_{FM}(t) &= \text{Re}\left\{A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}\, e^{j2\pi f_c t}\right\} \\ &= A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c + n f_m)t] \end{aligned} \quad (5.27)$$

Taking Fourier transform of Eq. (5.27), we get

$$S(f) = \frac{1}{2} A_c \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)] \quad (5.28)$$

From the spectral analysis we see that there is a carrier component and a number of side-frequencies around the carrier frequency at $\pm n f_m$.

The Bessel function may be expanded in a power series given by

$$J_n(\beta) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1}{2}\beta\right)^{n+2k}}{k!\,(k+n)!} \quad (5.29)$$

Plots of Bessel function $J_n(\beta)$ versus modulation index β for $n = 0, 1, 2, 3, 4$ are shown in Figure 5.3.

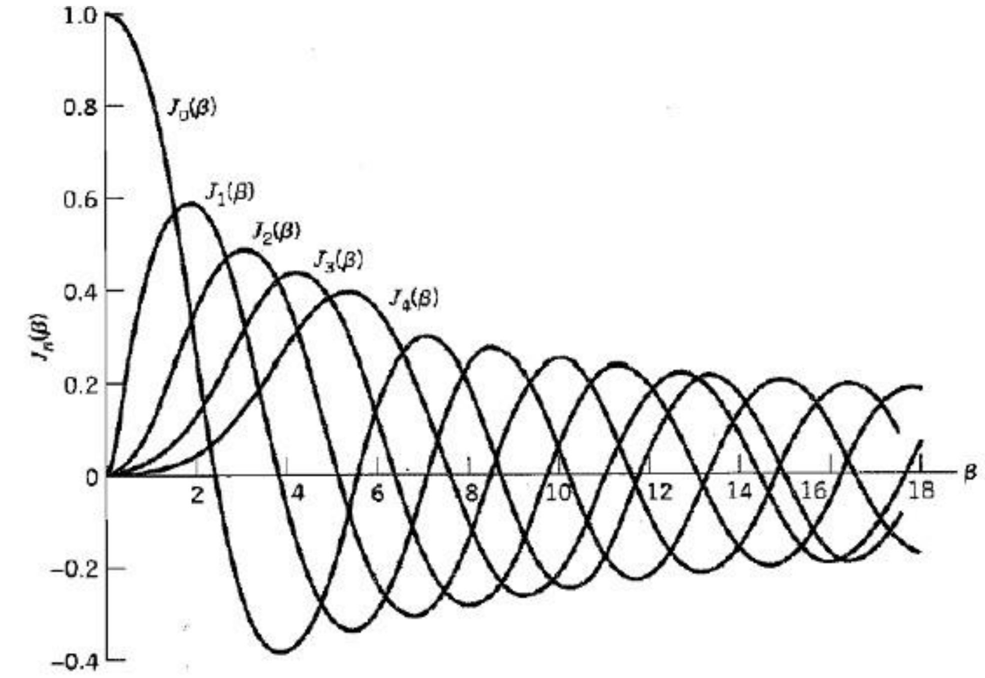


Figure 5.3 Plot of Bessel function as a function of modulation index.

Figure 5.3 shows that for any fixed value of n, the magnitude of $J_n(\beta)$ decreases as β increases. One property of Bessel function is that

$$J_{-n}(\beta) = \begin{cases} J_n(\beta), & n \text{ even} \\ -J_n(\beta), & n \text{ odd} \end{cases} \tag{5.30}$$

One more property of Bessel function is that

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1 \tag{5.31}$$

(iii) The average power of the FM wave remains constant. To prove this, let us determine the average power of Eq. (5.27) which is equal to

$$P = \frac{1}{2} A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

Using Eq. (5.31), the average power P is now

$$P = \frac{1}{2} A_c^2$$

TRANSMISSION BANDWIDTH OF FM WAVE

The transmission bandwidth of an FM wave depends on the modulation index þ. The modulation index, on the other hand, depends on the modulating amplitude and modulating frequency. It is almost impossible to determine the exact bandwidth of the FM wave. Rather, we use a rule-of-thumb expression for determining the FM bandwidth.

For single-tone frequency modulation, the approximated bandwidth is determined by the expression

$$B = 2(\Delta f + f_m) = 2(\beta + 1)f_m = 2\Delta f\left(1 + \frac{1}{\beta}\right)$$

This expression is regarded as the Carson's rule. The FM bandwidth determined by this rule accommodates at least 98 % of the total power.

For an arbitrary message signal n(t) with bandwidth or maximum frequency W, the bandwidth of the corresponding FM wave may be determined by Carson's rule as

$$B = 2(\Delta f + W) = 2(D + 1)W = 2\Delta f\left(1 + \frac{1}{D}\right)$$

GENERATION OF FM WAVES

FM waves are normally generated by two methods: indirect method and direct method.

Indirect Method (Armstrong Method) of FM Generation

The direct methods of generation of FM, LC oscillators are to be used. The crystal oscillator cannot be used. The LC oscillators are not stable enough for the communication or broadcast purpose. Thus, the direct methods cannot be used for the broadcast applications.
The alternative method is to use the indirect method called as the Armstrong method of FM generation.

In this method, the FM is obtained through phase modulation. A crystal oscillator can be used hence the frequency stability is very high and this method is widely used in practice.

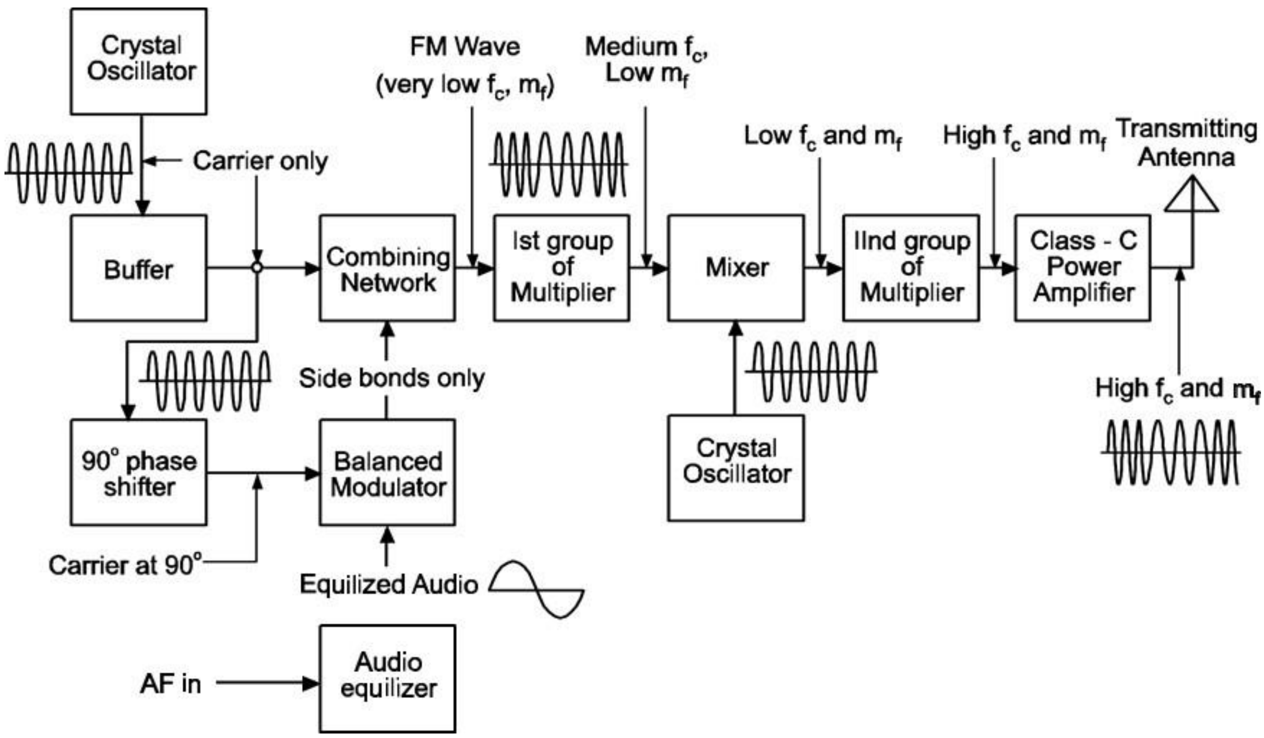


Fig 3.6: Indirect Method (Armstrong Method) of FM Generation

Working Principle

The working operation of this system can be divided into two parts as follows:

Part I: Generate a narrow band FM wave using a phase modulator.
Part II: Use the frequency multipliers and mixer to obtain the required values of frequency deviation, carrier and modulation index.

Part I: Generate a narrow band FM using Phase Modulator

As discussed carrier, we can generate FM using a phase modulator.

The modulating signal x(t) is passed through an integrator before applying it to the phase modulator as shown in figure 1.

Let the narrow band FM wave produced at the output of the phase modulator be represented by $s_1(t)$ i.e.,

$$s_1(t) = V_{c1} \cos [\, 2\pi f_1 t + \phi_1(t) \,]$$

where V_{c1} is the amplitude and f_1 is the frequency of the carrier produced by the crystal oscillator. The phase angle $\Phi_1(t)$ of $s_1(t)$ is related to x(t) as follows:

$$\phi_1(t) = 2\pi k_1 \int_0^t x(t)\, dt$$

where k_1 represents the frequency sensitivity of the modulator.
If $\Phi_1(t)$ is very small then,

$$\cos [\phi_1(t)] \cong 1 \quad \text{and} \quad \sin [\phi_1(t)] \cong \phi_1(t)$$

Hence, the approximate expression for $s_1(t)$ can be obtained as follows:

$$s_1(t) = V_{c1} \cos [2\pi f_1 t + \phi_1(t)]$$
$$= V_{c1} [\cos (2\pi f_1 t) \cos \phi_1(t) - \sin (2\pi f_1 t) \sin \phi_1(t)]$$

After approximation, we get,

$$s_1(t) = V_{c1} [\cos(2\pi f_1 t) \times 1 - \phi_1(t) \sin(2\pi f_1 t)$$
$$s_1(t) = V_{c1} \cos(2\pi f_1 t) - V_{c1}\phi_1(t) \sin(2\pi f_1 t)$$

Substituting,

$$\phi_1(t) = 2\pi k_1 \int_0^t x(t)\, dt, \quad \text{we obtain}$$

$$s_1(t) = V_{c1} \cos(2\pi f_1 t) - 2\pi k_1 V_{c1} \sin(2\pi f_1 t) \int_0^t x(t)\, dt$$

This expression represents a narrow band FM. Thus, at the output of the phase modulator, we obtain a narrow band FM wave.

Part II: Implementation of the Phase Modulator

shows the block diagram of phase modulator circuit.

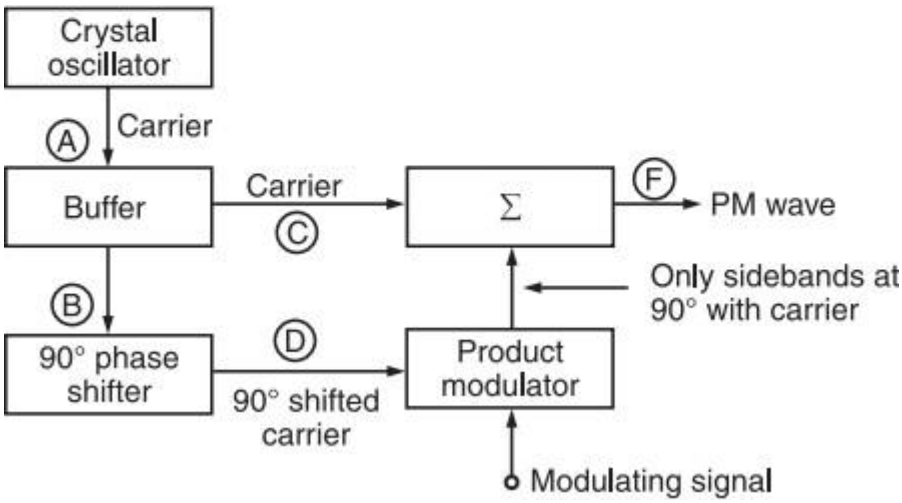


Fig.3.7: Phase Modulator Circuit

Working Principle

The crystal oscillator produces a stable unmodulated carrier which is applied to the 90° phase shifter as well as the combining network through a buffer.

The 90° phase shifter produces a 90° phase shifted carrier. It is applied to the balanced modulator along with the modulating signal.

Thus, the carrier used for modulation is 90° shifted with respect to the original carrier.

At the output of the product modulator, we get DSB SC signal i.e., AM signal without carrier.

This signal consists of only two sidebands with their resultant in phase with the 90° shifted carrier.

The two sidebands and the original carrier without any phase shift are applied to a combining network (∑). At the output of the combining network, we get the resultant of vector addition of the carrier and two sidebands as shown in figure 2.

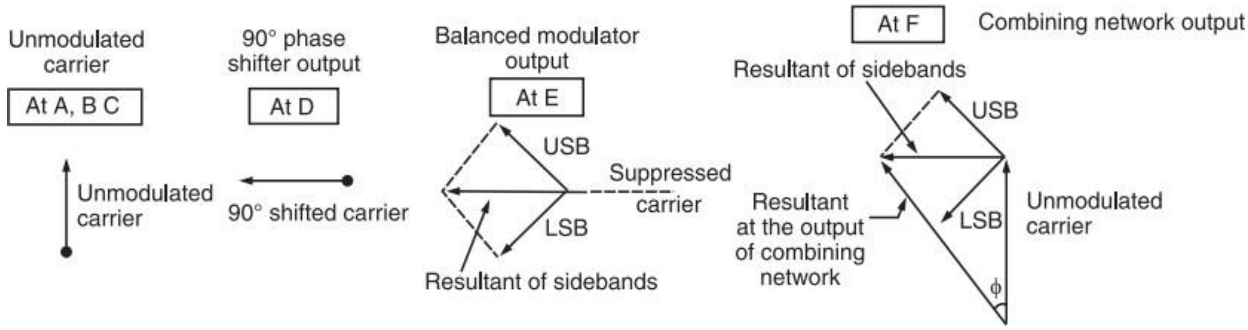


Fig.3.8: Phasors explaining the generation of PM

Now, as the modulation index is increased, the amplitude of sidebands will also increase. Hence, the amplitude of their resultant increases. This will increase the angle Φ made by the resultant with unmodulated carrier.

The angle Φ decreases with reduction in modulation index as shown in figure 3.

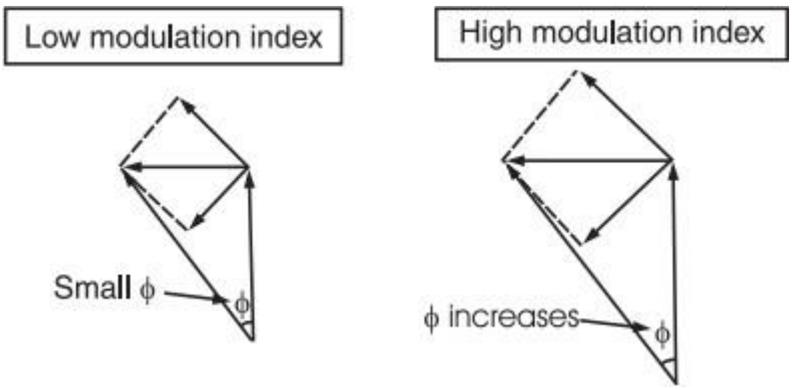


Fig.3.9: Effect of modulation index on frequency f

Thus, the resultant at the output of the combining network is phase modulated. Hence, the block diagram of figure.1 operates as a phase modulator.

Part III: Use of Frequency Multipliers Mixer and Amplifier

The FM signal produced at the output of phase modulator has a low carrier frequency and low modulation index. They are increased to an adequately high value with the help of frequency multipliers and mixer.

Direct Method of FM Generation

In this method, the instantaneous frequency $f(t)$ of the carrier signal c(t) is varied directly with the instantaneous value of the modulating signal n(t). For this, an oscillator is used in which any one of the reactive components (either C or L) of the resonant network of the oscillator is varied linearly with n(t). We can use a varactor diode or a varicap as a voltage-variable capacitor whose capacitance solely depends on the reverse-bias voltage applied across it. To vary such capacitance linearly with n(t), we have to reverse-bias the diode by the fixed DC voltage and operate within a small linear portion of the capacitance-voltage characteristic curve. The unmodulated fixed capacitance C_0 is linearly varied by n(t) such that the resultant capacitance becomes

$$C(t) = C_0 - kn(t)$$

where the constant k is the sensitivity of the varactor diode (measured in capacitance per volt).

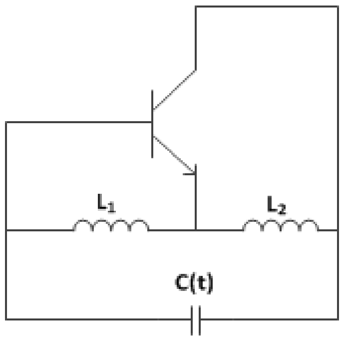


Fig3.10: Hartley oscillator for FM generation

The above figure shows the simplified diagram of the Hartley oscillator in which is implemented the above discussed scheme. The frequency of oscillation for such an oscillator is given

$$f(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)C(t)}}$$

$$f(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)(C_0 - km(t))}}$$

$$= \frac{1}{2\pi\sqrt{(L_1 + L_2)C_0}} \frac{1}{\sqrt{1 - \frac{km(t)}{C_0}}}$$

or, $$f(t) = f_c\left(1 - \frac{km(t)}{C_0}\right)^{-1/2}$$

where f_c is the unmodulated frequency of oscillation. Assuming,

$$\frac{km(t)}{C_0} \ll 1$$

we have from binomial expansion,

$$\left(1 - \frac{km(t)}{C_0}\right)^{-1/2} \approx 1 + \frac{km(t)}{2C_0}$$

$$f(t) \approx f_c\left(1 + \frac{km(t)}{2C_0}\right)$$

$$= f_c + \frac{kf_c m(t)}{2C_0}$$

$$\text{or}, f(t) = f_c + k_f m(t)$$

$$k_f = \frac{kf_c}{2C_0}$$

is the frequency sensitivity of the modulator. The Eq. (5.42) is the required expression for the instantaneous frequency of an FM wave. In this way, we can generate an FM wave by direct method.

Direct FM may be generated also by a device in which the inductance of the resonant circuit is linearly varied by a modulating signal n(t); in this case the modulating signal being the current.

The main advantage of the direct method is that it produces sufficiently high frequency deviation, thus requiring little frequency multiplication. But, it has poor frequency stability. A feedback scheme is used to stabilize the frequency in which the output frequency is compared with the constant frequency generated by highly stable crystal oscillator and the error signal is feedback to stabilize the frequency.

DEMODULATION OF FM WAVES

The process to extract the message signal from a frequency modulated wave is known as frequency demodulation. As the information in an FM wave is contained in its instantaneous frequency, the frequency demodulator has the task of changing frequency variations to amplitude variations. Frequency demodulation method is generally categorized into two types: direct method and indirect method. Under direct method category, we will discuss about limiter discriminator method and under indirect method, phase-locked loop (PLL) will be discussed.

Limiter Discriminator Method

Recalling the expression of FM signal,

$$s(t) = A_c \cos [2\pi f_c t + 2\pi k_f \int_0^t m(t)dt]$$

In this method, extraction of n(t) from the above equation involves the three steps: amplitude limit, discrimination, and envelope detection.

A. Amplitude Limit

During propagation of the FM signal from transmitter to receiver the amplitude of the FM wave (supposed to be constant) may undergo changes due to fading and noise. Therefore, before further processing, the amplitude of the FM signal is limited to reduce the effect of fading and noise by using limiter as discussed in the section 5.9. The amplitude limitation will not affect the message signal as the amplitude of FM does not carry any information of the message signal.

B. Discrimination/ Differentiation

In this step we differentiate the FM signal as given by

$$\frac{ds(t)}{dt} = \frac{d}{dt}\left\{A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right]\right\}$$

$$= \frac{d\left\{A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right]\right\}}{d\left\{2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right\}} \frac{d\left\{2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right\}}{dt}$$

$$= -A_c[2\pi f_c + 2\pi k_f m(t)] \sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right]$$

Here both the amplitude and frequency of this signal are modulated.

In this case, the differentiator is nothing but a circuit that converts change in frequency into corresponding change in voltage or current as shown in Fig.3.11. The ideal differentiator has transfer function

$$H(jw) = j2\pi f$$

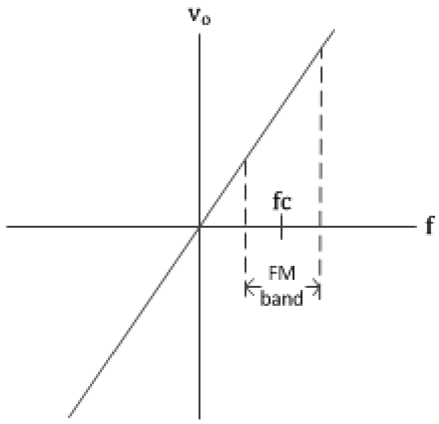


Figure 3.11: Transfer function of ideal differentiator.

Instead of ideal differentiator, any circuit can be used whose frequency response is linear for some band in positive slope. This method is known as slope detection. For this, linear segment with positive slope of RC high pass filter or LC tank circuit can be used. Figure 3.12 shows the use of an LC circuit as a differentiator. The drawback is the limited linear portion in the slope of the tank circuit. This is not suitable for wideband FM where the peak frequency deviation is high.

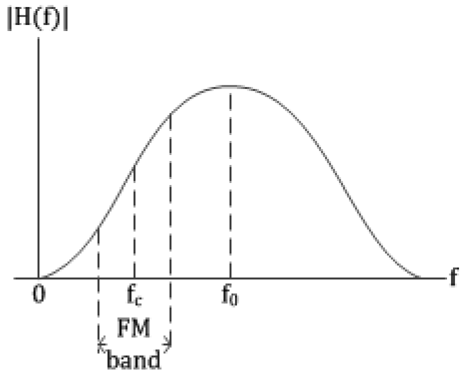


Figure 3.12: Use of LC tank circuit as a differentiator.

A better solution is the ratio or balanced slope detector in which two tank circuits tuned at $f_c + \Delta f$ and $f_c - \Delta f$ are used to extend the linear portion as shown in below figure.

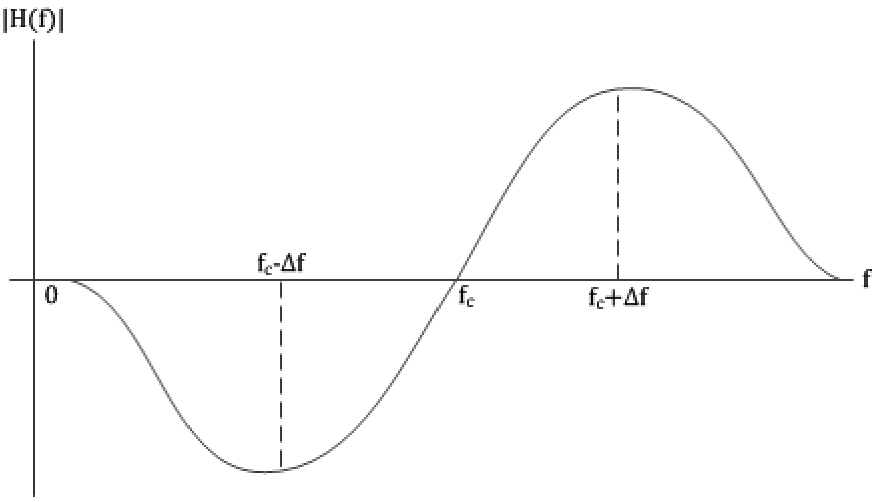


Figure 3.13: Frequency response of balanced slope detector.

Another detector called Foster-seely discriminator eliminates two tank circuits but still offer the same linear as the ratio detector.

C. Envelope Detection

The third step is to send the differentiated signal to the envelope detector to recover the message signal.

Phase-Locked Loop (PLL) as FM Demodulator

A PLL consists of a multiplier, a loop filter, and a VCO connected together to form a feedback loop as shown in Fig. 3.14. Let the input signal be an FM wave as defined by

$$s(t) = A_c \cos[2\pi f_c t + \emptyset_1(t)]$$

$$\emptyset_1(t) = 2\pi k_f \int_0^t m(t)dt$$

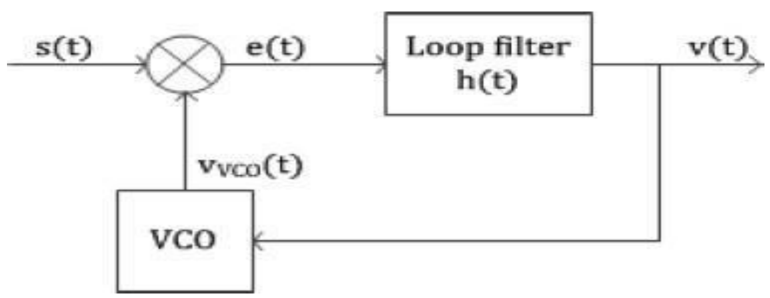


Fig 3.14: PLL Demodulator

Let the VCO output be defined by

$$v_{VCO}(t) = A_v \sin[2\pi f_c t + \emptyset_2(t)]$$

where

$$\emptyset_2(t) = 2\pi k_v \int_0^t v(t)dt$$

Here k_v is the frequency sensitivity of the VCO measured in hertz per volt. The multiplication of $s(t)$ and $v_{VCO}(t)$ results

$$s(t)v_{VCO}(t) = A_c \cos[2\pi f_c t + \emptyset_1(t)]\, A_v \sin[2\pi f_c t + \emptyset_2(t)]$$

$$= \frac{A_c A_v}{2} \sin[4\pi f_c t + \emptyset_1(t) + \emptyset_2(t)] + \frac{A_c A_v}{2} \sin[\emptyset_2(t) - \emptyset_1(t)]$$

The high-frequency component is removed by the low-pass filtering of the loop filter. Therefore, the input signal to the loop filter can be considered as

$$e(t) = \frac{A_c A_v}{2} \sin[\emptyset_2(t) - \emptyset_1(t)]$$

The difference $\emptyset_2(t) - \emptyset_1(t) = \emptyset_e(t)$ constitutes the phase error. Let us assume that the PLL is in phase lock so that the phase error is very small. Then,

$$\sin[\emptyset_2(t) - \emptyset_1(t)] \approx \emptyset_2(t) - \emptyset_1(t)$$

$$\emptyset_e(t) = 2\pi k_v \int_0^t v(t)dt - \emptyset_1(t)$$

$$e(t) = \frac{A_c A_v}{2} \emptyset_e(t)$$

Differentiating Eq. (5.48) with respect to time, we get

$$\frac{d\emptyset_e(t)}{dt} = 2\pi k_v v(t) - \frac{d\emptyset_1(t)}{dt}$$

Since

$$v(t) = e(t) * h(t) = \frac{A_c A_v}{2} [\emptyset_e(t) * h(t)]$$

Eq. (5.50) becomes

$$\frac{d\emptyset_e(t)}{dt} = 2\pi k_v \frac{A_c A_v}{2} [\emptyset_e(t) * h(t)] - \frac{d\emptyset_1(t)}{dt}$$

$$or, \pi k_v A_c A_v [\emptyset_e(t) * h(t)] - \frac{d\emptyset_e(t)}{dt} = \frac{d\emptyset_1(t)}{dt}$$

Taking Fourier transform of Eq. (5.52), we get

$$\pi k_v A_c A_v \emptyset_e(f) H(f) - j2\pi f \emptyset_e(f) = j2\pi f \emptyset_1(f)$$

$$or, \emptyset_e(f) = \frac{j2\pi f}{\pi k_v A_c A_v H(f) - j2\pi f} \emptyset_1(f)$$

$$or, \emptyset_e(f) = \frac{1}{\frac{\pi k_v A_c A_v}{j2\pi f} H(f) - 1} \emptyset_1(f)$$

Fourier transform of Eq. (5.51) is

$$V(f) = \frac{A_c A_v}{2} \emptyset_e(f) H(f)$$

Substituting Eq. (5.53) into (5.54), we get

$$V(f) = \frac{A_c A_v}{2} \frac{1}{\frac{\pi k_v A_c A_v}{j2\pi f} H(f) - 1} \emptyset_1(f) H(f)$$

We design $H(f)$ such that

$$\left| \frac{\pi k_v A_c A_v}{j2\pi f} H(f) \right| \gg 1$$

in the frequency band $|f| < W$ of the message signal.

$$\therefore V(f) = \frac{A_c A_v}{2} \frac{1}{\frac{\pi k_v A_c A_v}{j2\pi f} H(f)} \emptyset_1(f) H(f)$$

$$or, V(f) = \frac{1}{2\pi k_v} j2\pi f \emptyset_1(f)$$

Taking inverse Fourier transform of Eq. (4.56), we get

$$v(t) = \frac{1}{2\pi k_v} \frac{d\emptyset_1(t)}{dt}$$

$$= \frac{1}{2\pi k_v} \frac{d}{dt} \left\{ 2\pi k_f \int_0^t m(t) dt \right\}$$

$$= \frac{1}{2\pi k_v} 2\pi k_f m(t)$$

$$\therefore v(t) = \frac{k_f}{k_v} m(t)$$

Since the control voltage of the VCO is proportional to the message signal, $v(t)$ is the demodulated signal.

We observe that the output of the loop filter with frequency response $H(f)$ is the desired message signal. Hence the bandwidth of $H(f)$ should be the same as the bandwidth W of the message signal. Consequently, the noise at the output of the loop filter is also limited to the bandwidth W. On the other hand, the output from the VCO is a wideband FM signal with an instantaneous frequency that follows the instantaneous frequency of the received FM signal.

PRE-EMPHASIS AND DE-EMPHASIS NETWORKS

In FM, the noise increases linearly with frequency. By this, the higher frequency components of message signal are badly affected by the noise. To solve this problem, we can use a preemphasis filter of transfer function $H_p(f)$ at the transmitter to boost the higher frequency components before modulation. Similarly, at the receiver, the deemphasis filter of transfer function $H_d(f)$can be used after demodulator to attenuate the higher frequency components thereby restoring the original message signal.

The preemphasis network and its frequency response are shown in Figure 3.15 (a) and (b) respectively. Similarly, the counter part for deemphasis network is shownin Figure 3.16.

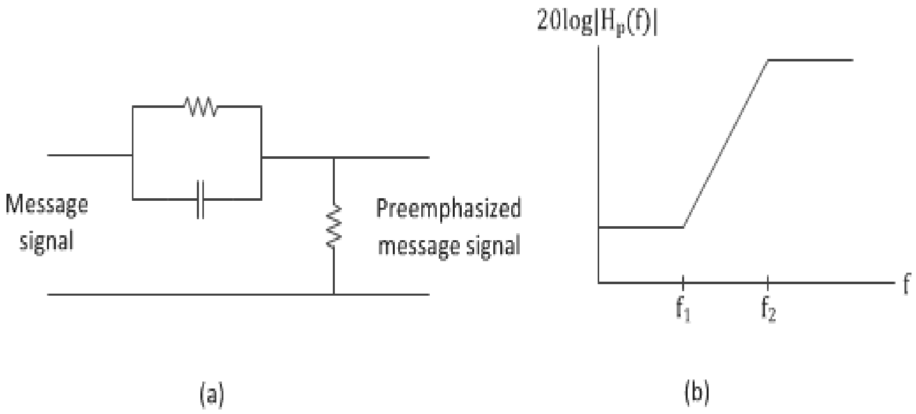


Figure 3.15 ;(a) Preemphasis network. (b) Frequency response of preemphasis network.

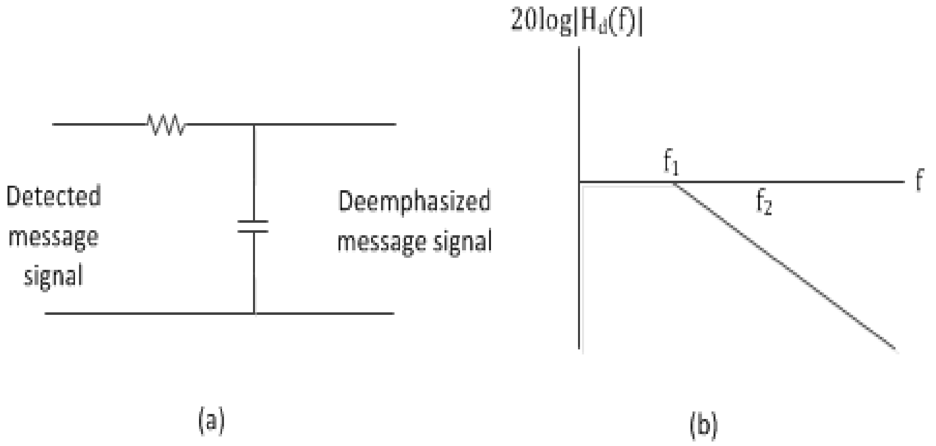


Figure 3.16 (a) De-emphasis network. (b) Frequency response of De-emphasis network.

In FM broadcasting, f_1 and f_2 are normally chosen to be 2.1 kHz and 30 kHz Respectively.

The frequency response of pre-emphasis network is

$$H_p(f) = \left(\frac{w_2}{w_1}\right)\frac{jw + w_1}{jw + w_2}$$

Here, $w = 2\pi f$ and $w_1 = 2\pi f_1$. For $w \ll w_1$,

$$H_p(f) \approx 1$$

And for $w_1 \ll w \ll w_2$,

$$H_p(f) \approx \frac{j2\pi f}{w_1}$$

So, the amplitude of frequency components less than 2.1 kHz are left unchanged and greater than that are increased proportional to f.

The frequency response of deemphasis network is

$$H_d(f) = \frac{w_1}{j2\pi f + w_1}$$

For $w \ll w_2$,

$$H_p(f) \approx \frac{j2\pi f + w_1}{w_1}$$

such that

$$H_p(f)H_d(f) \approx 1$$

over the baseband of 0 to 15 KHz.

Comparison of AM and FM:

S.NO	AMPLITUDE MODULATION	FREQUENCY MODULATION
1.	Band width is very small which is one of the biggest advantage	It requires much wider channel (7 to 15 times) as compared to AM.
2.	The amplitude of AM signal varies depending on modulation index.	The amplitude of FM signal is constant and independent of depth of the modulation.
3.	Area of reception is large	The are of reception is small since it is limited to line of sight.
4.	Transmitters are relatively simple & cheap.	Transmitters are complex and hence expensive.
5.	The average power in modulated wave is greater than carrier power. This added power is provided by modulating source.	The average power in frequency modulated wave is same as contained in un-modulated wave.
6.	More susceptible to noise interference and has low signal to noise ratio, it is more difficult to eliminate effects of noise.	Noise can be easily minimized amplitude variations can be eliminated by using limiter.
7.	it is not possible to operate without interference.	it is possible to operate several independent transmitters on same frequency.
8.	The maximum value of modulation index = 1, other wise over-modulation would result in distortions.	No restriction is placed on modulation index.

FM Transmitter

The FM transmitter is a single transistor circuit. In the telecommunication, the frequency modulation (FM)transfers the information by varying the frequency of carrier wave according to the message signal. Generally, the FM transmitter uses VHF radio frequencies of 87.5 to 108.0 MHz to transmit & receive the FM signal. This transmitter accomplishes the most excellent range with less power. The performance and working of the wireless audio transmitter circuit is depends on the induction coil & variable capacitor. This article will explain about the working of the FM transmitter circuit with its applications.

The FM transmitter is a low power transmitter and it uses FM waves for transmitting the sound, this transmitter transmits the audio signals through the carrier wave by the difference of frequency. The carrier wave frequency is equivalent to the audio signal of the amplitude and the FM transmitter produce VHF band of 88 to 108MHZ.Plese follow the below link for: Know all About Power Amplifiers for FM Transmitter

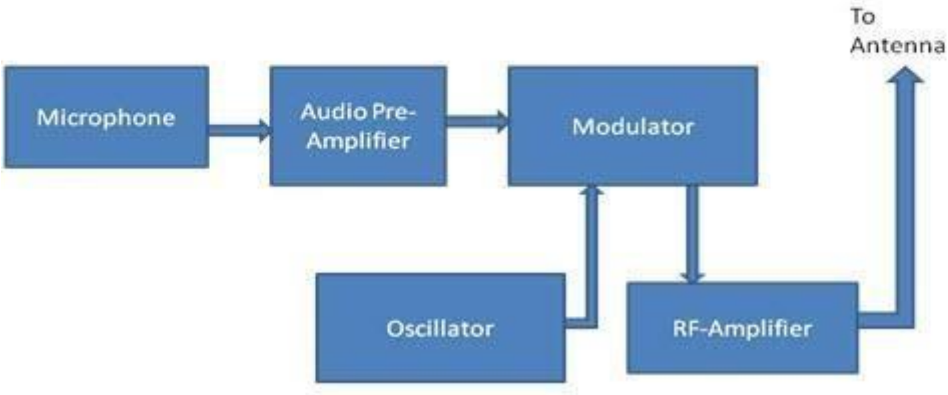


Fig 3.17: Block Diagram of FM Transmitter

Working of FM Transmitter Circuit

The following circuit diagram shows the FM transmitter circuit and the required electrical and electronic components for this circuit is the power supply of 9V, resistor, capacitor, trimmer capacitor, inductor, mic, transmitter, and antenna. Let us consider the microphone to understand the sound signals and inside the mic there is a presence of capacitive sensor. It produces according to the vibration to the change of air pressure and the AC signal.

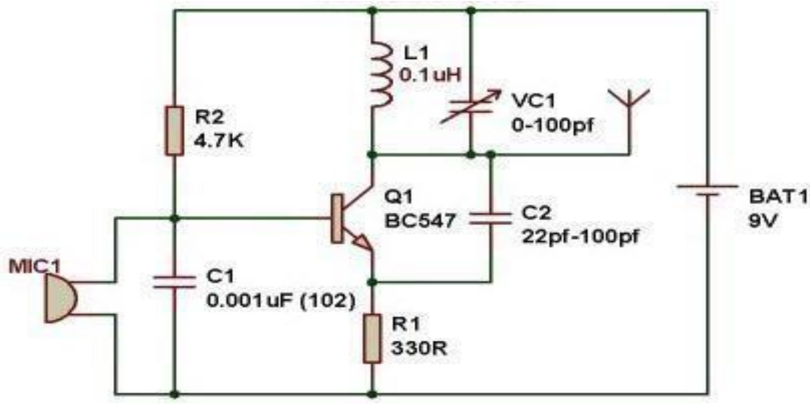


Fig 3.18:FM Transmitter circuit

The formation of the oscillating tank circuit can be done through the transistor of 2N3904 by using the inductor and variable capacitor. The transistor used in this circuit is an NPN transistor used for general purpose amplification. If the current is passed at the inductor L1 and variable capacitor then the tank circuit will oscillate at the resonant carrier frequency of the FM modulation. The negative feedback will be the capacitor C2 to the oscillating tank circuit.

To generate the radio frequency carrier waves the FM transmitter circuit requires an oscillator. The tank circuit is derived from the LC circuit to store the energy for oscillations.

The input audio signal from the mic penetrated to the base of the transistor, which modulates the LC tank circuit carrier frequency in FM format. The variable capacitor is used to change the resonant frequency for fine modification to the FM frequency band. The modulated signal from the antenna is radiated as radio waves at the FM frequency band and the antenna is nothing but copper wire of 20cm long and 24 gauge. In this circuit the length of the antenna should be significant and here you can use the 25-27 inches long copper wire of the antenna.

Application of Fm Transmitter

- The FM transmitters are used in the homes like sound systems in halls to fill the sound with the audio source.
- These are also used in the cars and fitness centers.
- The correctional facilities have used in the FM transmitters to reduce the prison noise in common areas.

Advantages of the FM Transmitters

- The FM transmitters are easy to use and the price is low
- The efficiency of the transmitter is very high
- It has a large operating range
- This transmitter will reject the noise signal from an amplitude variation.

UNIT IV
RADIO RECEIVERS NOISE

- **Radio Receiver:** Introduction to radio receivers & its parameters–
- Super heterodyne AM & FM Receiver.
- **Noise:** Review of noise
- noise sources
- noise figure
- Performance analysis of AM, DSB-SC, SSB-SC in the presence of noise
- Illustrative Problems.

Introduction To Radio Receivers:

In radio communications, a radio receiver is an electronic device that receives radio waves and converts the information carried by them to a usable form. The antenna intercepts radio waves (electromagnetic waves) and converts them to tiny alternating currents which are applied to the receiver and the receiver extracts the desired information. The receiver uses electronic filters to separate the desired radio frequency signal from all the other signals picked up by the antenna, an electronic amplifier to increase the power of the signal for further processing, and finally recovers the desired information through demodulation. The information produced by the receiver may be in the form of sound, moving images (television), or data. Radio receivers are very widely used in modern technology, as components of communications, broadcasting, remote control, and wireless networking systems.

Receiver Characteristics

The performance of the radio receiver can be measured in terms of following receiver characteristics

- Selectivity
- Sensitivity
- Fidelity
- Image frequency and its rejection
- Double Spotting

Selectivity

The ability of the receiver to select the wanted signals among the various incoming signals is termed as Selectivity. It rejects the other signals at closely lying frequencies. Selectivity of a receiver changes with incoming signal frequency and are poorer at high frequencies.

Selectivity in a receiver is obtained by using tuned circuits. These are LC circuits tuned to resonate at a desired signal frequency. The Q of these tuned circuits determines the selectivity. Selectivity shows the attenuation that the receiver offers to signals at frequencies near to the one to which it is tuned. A good receiver isolates the desired signal in the RF spectrum and eliminates all other signals.

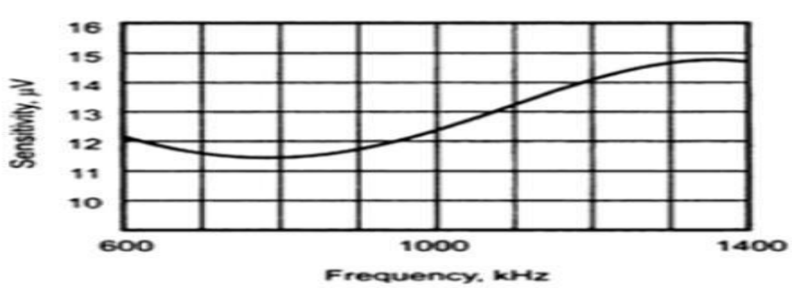


Fig 4.1: Sensitivity

Sensitivity

The sensitivity of a radio receiver is its ability to amplify weak signals. It is often defined in terms of the voltage that must be applied to the receiver input terminals to give a standard output power, measured at the output terminals. The most important factors determining the sensitivity of a super heterodyne receiver are the gain of the IF amplifier(s) and that of the RF amplifier .The more gain that a receiver has, the smaller the input signal necessary to produce thedesired output power. Therefore sensitivity is a primary function of the overall receiver gain. Good communication receiver has a sensitivity of 0.2 to 1 µV

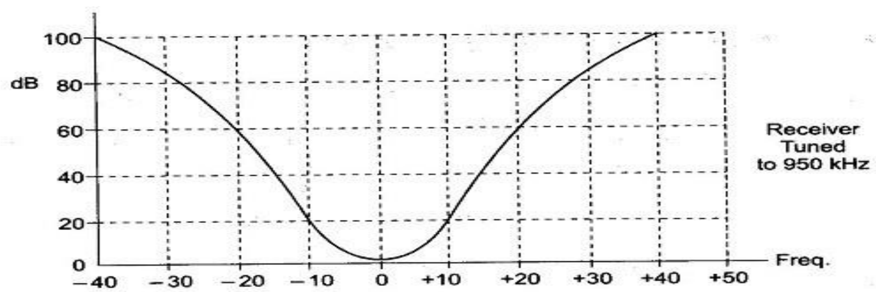


Fig 4.2: Selectivity

Fidelity

Fidelity refers to the ability of the receiver to reproduce all the modulating frequencies equally. Figure shows the typical fidelity curve for radio receiver.

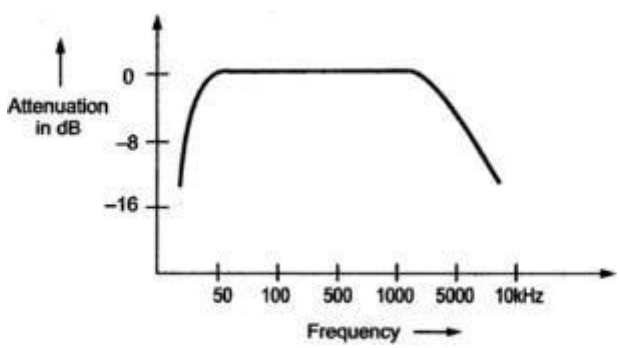


Fig.4.3. Typical Fidelity curve

The fidelity at the lower modulating frequencies is determined by the low frequency response of the IF amplifier and the fidelity at the higher modulating frequencies is determined by the high frequency response of the IF amplifier. Fidelity is difficult to obtain in AM receiver because good fidelity requires more bandwidth of IF amplifier resulting in poor selectivity.

Image frequency and its Rejection

In a standard broadcast receiver the local oscillator frequency is made higher than the incoming signal frequency for reasons that will become apparent .It is made equal at all times to the signal frequency plus the intermediate frequency. Thus f0=fs+fi or f0=fs−fi, no matter what the signal frequency may be. When f0 and fs are mixed, the difference frequency, which is one of the by-products, equal to fi is passed and amplified by the IF stage. If a frequency fsi manages to reach the mixer, such that fsi=fo+fi, that is , fsi=fs+2fi then this frequency will also produce fi when mixed with f0.

Unfortunately, this spurious intermediate-frequency signal will also be amplified by the IF stage and will therefore provide interference. This has the effect of two stations being receivedsimultaneously and is naturally undesirable. The term fsi is called image frequency and is definedas the signal frequency plus twice the intermediate frequency.

The rejection of an image frequency by a single –tuned circuit, i.e., the ratio of the gain at the signal frequency to the gain at the image frequency, is given by

$$\alpha= \sqrt{1+Q^2\rho^2}$$

where

Q=loaded Qoftuned circuit

If the receiver has an RF stage, then there are two tuned circuits , both tuned to fs .The rejection of each will be calculated by the same formula , and the total rejection will be product of the two.

Image rejection depends on the front-end selectivity of the receiver and must be achieved before the IF stage.Once the spurious frequency enters the first IF amplifier, it becomes impossible to remove it from the wanted signal.

Double spotting

This is well-known phenomenon, which manifests itself by the picking up of the same shortwave station at two nearby points on the receiver dial.It is caused by poor front-end selectivity, i.e., inadequate image-frequency rejection .That is to say, the front end of the receiver does not select different adjacent signals very well, but the IF stage takes care of eliminating almost all of them.

As a matter of interest, double spotting may be used to calculate the intermediate frequency of an unknown receiver, since the spurious point on the dial is precisely 2f, below the correct frequency .An improvement in image-frequency rejection will produce a corresponding reduction in double spotting.

Super heterodyne Receiver:

To solve basic problem of TRF receivers, first all the incoming RF frequencies are converted to fix lower frequency called Intermediate Frequency (IF).Then this fix intermediate frequency is amplified and detected to reproduce the original information. Since the characteristics of the IF amplifier are independent of the frequency to which the receiver is tuned, the selectivity and sensitivity of super heterodyne receivers are fairly uniform throughout its tuning range.

The basic concept and theory behind the super heterodyne radio involves the process of mixing. This enables signals to be translated from one frequency to another. The input frequency is often referred to as the RF input, whilst the locally generated oscillator signal is referred to as the local oscillator, and the output frequency is called the intermediate frequency as it is between the RF and audio frequencies.

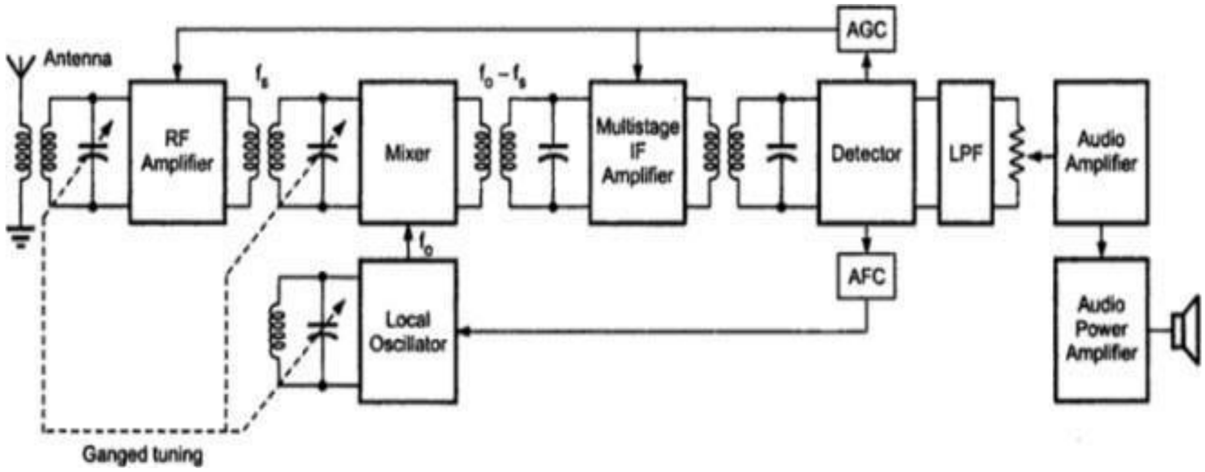


Fig4.4 .Block diagram of a Superheterodyne receiver

Operation:

Signals enter the receiver from the antenna and are applied to the RF amplifier where they are tuned to remove the image signal and also reduce the general level of unwanted signals on other frequencies that are not required.

The signals are then applied to the mixer along with the local oscillator where the wanted signal is converted down to the intermediate frequency. Here significant levels of amplification are applied and the signals are filtered. This filtering selects signals on one channel against those on the next. It is much larger than that employed in the front end.The advantage of the IF filter as opposed to RF filtering is that the filter can be designed for a fixed frequency. This allows for much better tuning. Variable filters are never able to provide the same level of selectivity that can be provided by fixed frequency ones.

Once filtered the next block in the superheterodyne receiver is the demodulator. This could be for amplitude modulation, single sideband, frequency modulation, or indeed any form of modulation. It is also possible to switch different demodulators in according to the mode being received.

The final element in the superheterodyne receiver block diagram is shown as an audio amplifier, although this could be any form of circuit block that is used to process or amplified the demodulated signal.

Another important circuit in the superheterodyne receiver is AGC and AFC circuit. AGC is used to maintain a constant output voltage level over a wide range of RF input signal levels.

It derives the dc bias voltage from the output of detector which is proportional to the amplitude of the received signal.This dc bias voltage is feedback to the IF amplifiers to control the gain of the receiver. As a result, it provides a constant output voltage level over a wide range of RF input signal levels. AFC circuit generated AFC signal which is used to adjust and stabilize the frequency of the local oscillator.

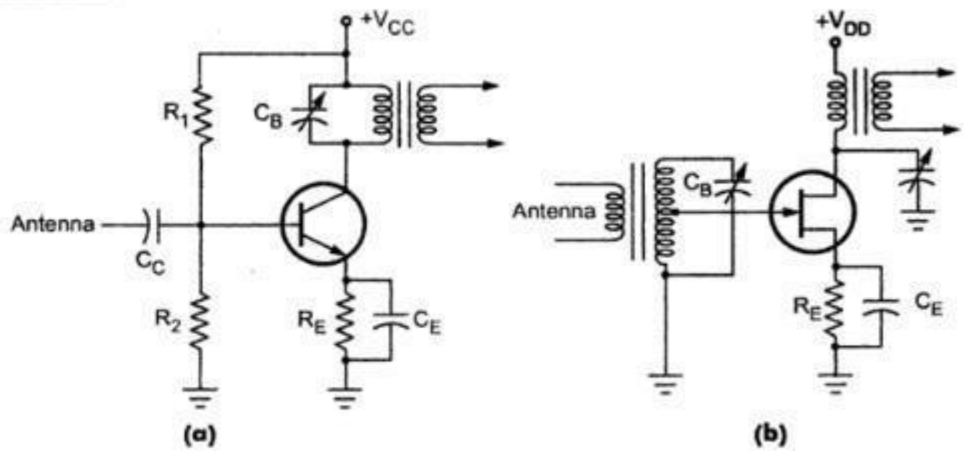
Advantages of the superheterodyne receiver

• IF stage permits use at very high frequencies.
• Because many components operate at the fixed IF, they can be optimized.
• Less expensive.
• Better selectivity
• Improved circuit stability.
• Uniform gain over a wide range of frequencies

Receiver Sections:

<u>RF tuning & amplification</u>:

RF amplifier provides initial gain and selectivity. RF amplifier is a simple class A circuit. This RF stage within the overall block diagram for the receiver provides initial tuning to remove the image signal. If noise performance for the receiver is important, then this stage will be designed for optimum noise performance. This RF amplifier circuit block will also increase the signal level so that the noise introduced by later stages is at a lower level in comparison to the wanted signal. A typical bipolar circuit as (a) and FET circuit as (b) is shown below.



Values of resistors R1 and R2 in the bipolar circuit are adjusted such that the amplifier works as a class A amplifier.The antenna is connected through coupling capacitor to the base of the transistor.This makes the circuit very broad band as the transistor will amplify virtually any signal picked up by the antenna. The collector is tuned with a parallel resonant circuit to provide the initial selectivity for the mixer input.

FET circuit Fig.(b) is more effective than the transistor circuit. Their high input impedance minimizes the loading on tuned circuits, thereby permitting the Q of the circuit to be higher and selectivity to be sharper.

<u>Local oscillator</u>:

The local oscillator circuit block can take a variety of forms. Early receivers used free running local oscillators. Today most receivers use frequency synthesizers, normally based around phase locked loops. These provide much greater levels of stability and enable frequencies to be programmed in a variety of ways.

Mixer or Frequency Changer or Converter:

Real-life mixers produce a variety of other undesired outputs, including noise and they may also suffer overload when very strong signals are present.
Although very basic non-linear devices can actually perform a basic RF mixing or multiplication process, the performance will be far from the ideal, and where good receiver performance is required, the specification of the RF mixer must be match this expectation.

The basic process of RF mixing or multiplication where the incoming RF signal and a local oscillator are mixed or multiplied together to produce signals at the sum and difference frequencies is key to the whole operation of the superheterodyne receiver.

There are a number of considerations when looking at the receiver design and topology with respect to the RF mixer.There are many different forms of mixer that can be used, and the choice of the type depends very much upon the receiver and the anticipated performance.

Separately Excited Mixer

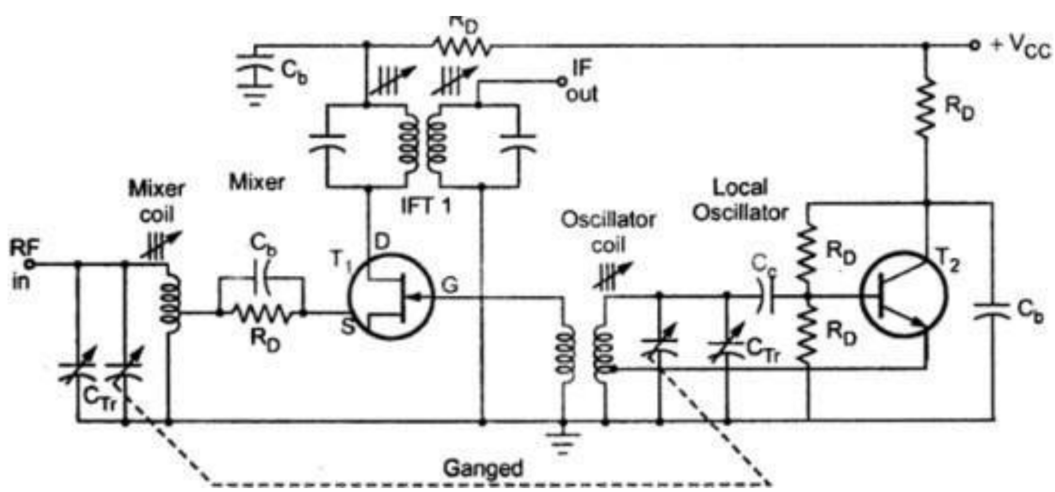


Fig.4.5 Separately Excited Mixer

In Separately excited mixer, one device acts as a mixer while the other supplies the necessary oscillations. Bipolar transistor T2 forms the Hartley oscillator circuit and oscillates with local frequency. FET T1 is a mixer whose gate s fed with the output of local oscillator and its bias is adjusted. The local oscillator varies the gate bias of the FET to vary its transconductance resulting intermediate frequency at the output. Output is taken through double tuned transformer in the drain of the mixer and fed to the IF amplifier.

Self Excited Mixer

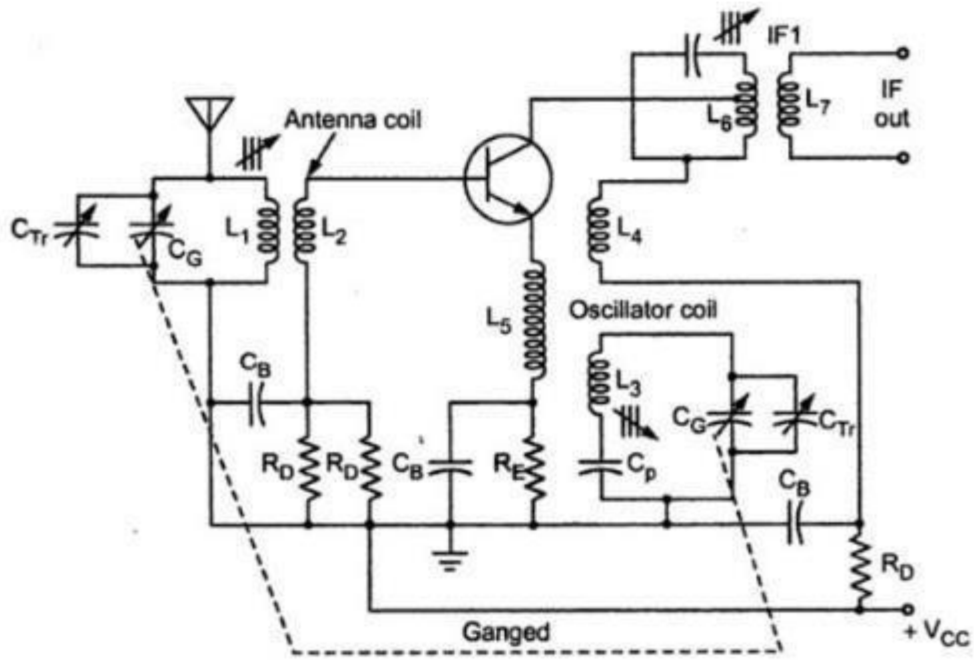


Fig 4.6: Self Excited Mixer

Self Excited circuit oscillates and the transconductance of the transistor is varied in a nonlinear manner at the local oscillator gate. This variable transconductance is used by the transistor to amplify the incoming RF signal.

IF amplifier & filter:

Tracking

The Superheterodyne receiver has number of tunable circuits which must all be tuned correctly if any given station is to be received. The ganged tuning is employed which mechanically couples all tuning circuits so that only one tuning control is required. Usually there are three tuned circuits: Antenna or RF tuned circuit, Mixer tuned circuit and local oscillator tuned circuit.

All these circuits must be tuned to get proper RF input and to get IF frequency at the output of the mixer. The process of tuning circuit to get the desired output is called Tracking. Tracking error will result in incorrect frequency being fed to the IF amplifier and these must be avoided.

To avoid tracking errors,ganged capacitors with identical sections are used.

A different value of inductance and capacitors called trimmers and padders are used to adjust the capacitance of the oscillator to the proper range. Common methods used for tracking are

- Padder Tracking
- Trimmer Tracking
- Three-Point Tracking

Intermediate IF amplifier:

Figure shows two Stage IF Amplifier. Two stages are transformer coupled and all IF transformers are single tuned i.e, tuned for single frequency.

IF amplifiers are tuned voltage amplifiers which istuned for the fixed frequency. Its function is to amplify only tuned frequency signal and reject all others .Most of the receiver gain is provided by the IF amplifiers and the required gain is obtained usually by two or more stages of IF amplifiers are required.

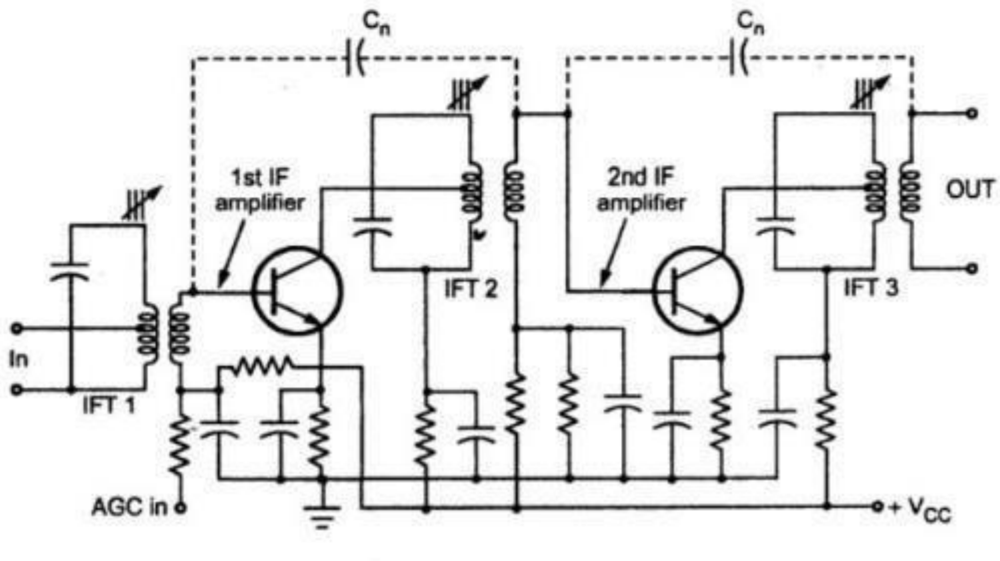


Fig 4.7Two Stage IF Amplifier

Automatic Gain Control, AGC:

An automatic gain control is incorporated into most superheterodyne radios. Its function is to reduce the gain for strong signals so that the audio level is maintained for amplitude sensitive forms of modulation, and also to prevent overloading. It is a system in which the overall gain of a radio receiver is varied automatically with the variations in the strength of the receiver signal to maintain the output substantially constant.

When the average signal level increases, the size of the AGC bias increases and the gain is deceased. When there is no signal, there is a minimum AGC bias and the amplifiers produce maximum gain. There are two types of AGC circuits. They are Simple AGC and Delayed AGC.

Simple AGC

In Simple AGC, the AGC bias starts to increase as soon as the received signal level exceeds the background noise level.As a result receiver gain starts falling down, reducing the sensitivity of the receiver.

In the circuit, the dc bias produced by half wave rectifier is used to control the gain of RF or IF amplifier. The time constant of the filter is kept at least 10 times longer than the period of the lowest modulation frequency received. If the time constant is kept longer, it will give better filtering. The recovered signal is then passes through capacitor to remove dc. The resulting ac signal is further amplified and applied to the loud speaker.

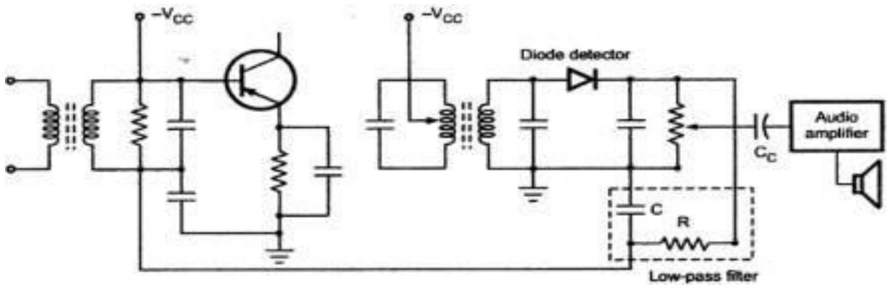


Fig 4.8 Simple AGC circuit

Delayed AGC

In simple AGC, the unwanted weak signals (noise signals) are amplified with high gain. To avoid this, in delayed AGC circuits, AGC bias is not applied to amplifiers until signal strength has reached a predetermined level, after which AGC bias is applied as with simple AGC, but more strongly.

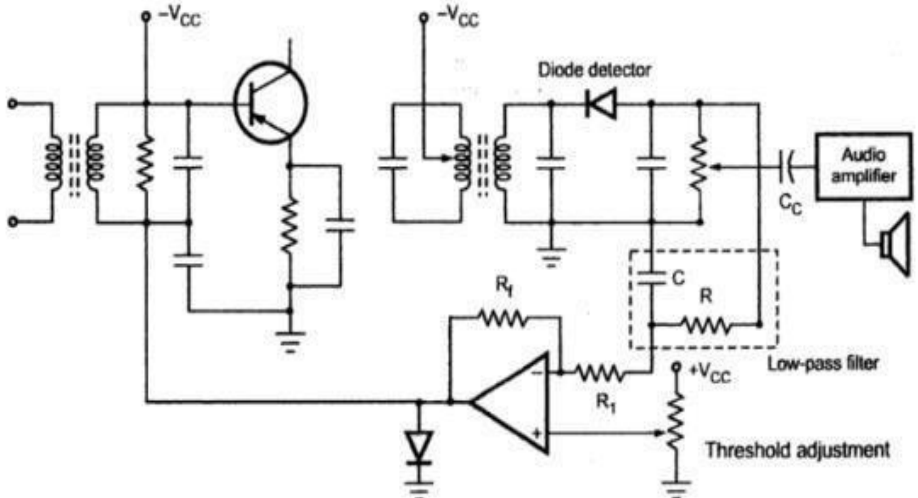


Fig 4.9.Delayed AGC circuit

AGC output is applied to the difference amplifier. It gives dc AGC only when AGC output generated by diode detector is above certain dc threshold voltage. This threshold voltage can be adjusted by adjusting the voltage at the positive input of the operational amplifier.

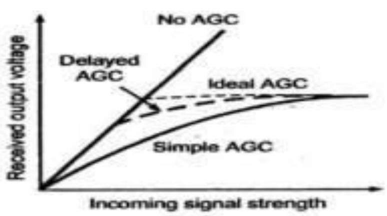


Fig.4.10 Response of receiver with various AGC

<u>Demodulator:</u>

The superheterodyne receiver block diagram only shows one demodulator, but in realit radios may have one or more demodulators dependent upon the type of signals being receiver.

Audio amplifier:

Once demodulated, the recovered audio is applied to an audio amplifier block to be amplified to the required level for loudspeakers or headphones. Alternatively the recovered modulation may be used for other applications whereupon it is processed in the required way bya specific circuit block.

Noise in communication system

- In electrical terms, noise may be defined as an unwanted form of energy that tends to interfere with the proper reception and reproduction of transmitted signals.
- For example, in receivers, several electrical disturbances produce noise and thus modifying the required signal in an unwanted form.
- In the case of radio receivers, noise may produce hiss-type sound in the output of loud speakers.
- Similarly, in T.V. receivers, noise may produce ‘snow’ which becomes superimposed on the picture output.
- In pulse communications, noise may produce unwanted pulses or cancel the required pulses.
- In other words, we can say that noise may limit the performance of a communication system.
- Noise is unwanted signal that affects wanted signal
- Noise is random signal that exists in communication systems

Effect of noise

- ➢ Degrades system performance (Analog and digital)
- ➢ Receiver cannot distinguish signal from noise
- ➢ Efficiency of communication system reduces

Types of noise

- ➢ Thermal noise/white noise/Johnson noise or fluctuation noise
- ➢ Shot noise
- ➢ Noise temperature
- ➢ Quantization noise

Noise temperature

Equivalent noise temperature is not the physical temperature of amplifier, but a theoretical construct, that is an equivalent temperature that produces that amount of noise power

$$T_e = (F - 1)$$

$$x(t) = n(t)\text{co}2\pi f_C t + n^{\wedge}(t)\sin 2\pi f_C t$$ \

and

$$y(t) = n(t)\cos 2\pi f_C t - n^{\wedge}(t)\sin 2\pi f_C t$$

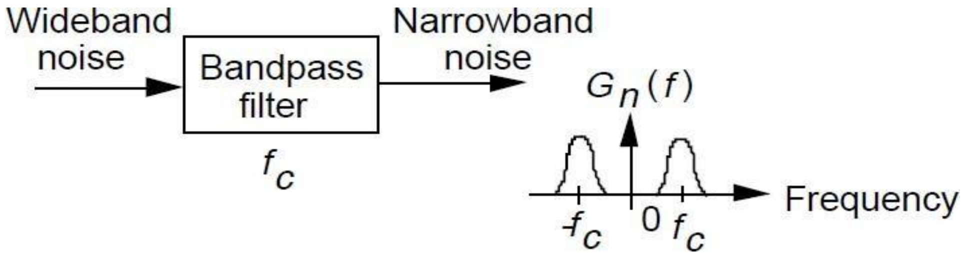


Fig4.11: generation of narrow band noise

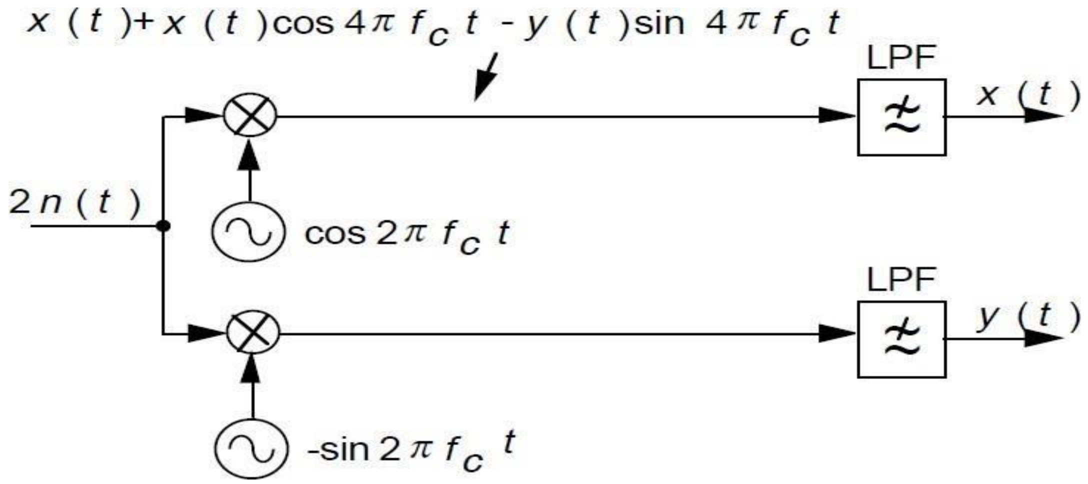


Fig 4.10: Generation of quadrature components of $n(t)$.

- Filters at the receiver have enough bandwidth to pass the desired signal but not too big to pass excess noise.
- Narrowband (NB) fc center frequency is much bigger that the bandwidth.
- Noise at the output of such filters is called narrowband noise (NBN).
- NBN has spectral concentrated about some mid-band frequency f_C
- The sample function of such NBN $n(t)$ appears as a sine wave of frequency f_C which modulates slowly in amplitude and phase

Input signal-to-noise ratio (SNR_I): is the ratio of the average power of modulated signal $s(t)$ to the average power of the filtered noise.

Output signal-to-noise ratio (SNR_O): is the ratio of the average power of demodulated message to the average power of the noise, both measured at the receiver output.

Channel signal-to-noise ratio (SNR_C): is the ratio of the average power of modulated signal $s(t)$ to the average power of the noise in the message bandwidth, both measured at the receiver input.

Noise figure

The Noise figure is the amount of noise power added by the electronic circuitry in the receiver to the thermal noise power from the input of the receiver. The thermal noise at the input to the receiver passes through to the demodulator. This noise is present in the receive channel and cannot be removed. The noise figure of circuits in the receiver such as amplifiers and mixers, adds additional noise to the receive channel. This raises the noise floor at the demodulator.

$$Noise\ Figure = \frac{Signal\ to\ noise\ ratio\ at\ input}{Signal\ to\ noise\ ratio\ at\ output}$$

Noise Bandwidth

A filter's equivalent noise bandwidth (ENBW) is defined as the bandwidth of a perfect rectangular filter that passes the same amount of power as the cumulative bandwidth of the channel selective filters in the receiver. At this point we would like to know the noise floor in our receiver, i.e. the noise power in the receiver intermediate frequency (IF) filter bandwidth that comes from kTB. Since the units of kTB are Watts/ Hz, calculate the noise floor in the channel bandwidth by multiplying the noise power in a 1 Hz bandwidth by the overall equivalent noise bandwidth in Hz.

NOISE IN DSB-SC SYSTEM:

Let the transmitted signal is

$$u(t) = A_c m(t)\cos(2\pi f_c t)$$

The received signal at the output of the receiver noise- limiting filter : Sum of this signal and filtered noise .A filtered noise process can be expressed in terms of its in-phase and quadrature components as

$$\begin{aligned} n(t) &= A(t)\cos[2\pi f_c t + \theta(t)] = A(t)\cos\theta(t)\cos(2\pi f_c t) - A(t)\sin\theta(t)\sin(2\pi f_c t) \\ &= n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t) \end{aligned}$$

where $n_C(t)$ is in-phase component and $n_S(t)$ is quadrature component

Received signal (Adding the filtered noise to the modulated signal)

$$r(t) = u(t) + n(t)$$
$$= A_c m(t)\cos(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

Demodulate the received signal by first multiplying r(t) by a locally generated sinusoid cos(2□ f_ct + ϕ), where □is the phase of the sinusoid.Then passing the product signal through an ideal lowpass filter having a bandwidth W.

The multiplication of r(t) with cos(2πfct + ϕ) yields

$$\begin{aligned}
&r(t)\cos(2\pi f_c t + \phi) \\
&\quad = u(t)\cos(2\pi f_c t + \phi) + n(t)\cos(2\pi f_c t + \phi) \\
&\quad = A_c m(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \phi) \\
&\qquad + n_c(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \phi) - n_s(t)\sin(2\pi f_c t)\cos(2\pi f_c t + \phi) \\
&\quad = \tfrac{1}{2} A_c m(t)\cos(\phi) + \tfrac{1}{2} A_c m(t)\cos(4\pi f_c t + \phi) \\
&\qquad + \tfrac{1}{2}\left[n_c(t)\cos(\phi) + n_s(t)\sin(\phi)\right] + \tfrac{1}{2}\left[n_c(t)\cos(4\pi f_c t + \phi) - n_s(t)\sin(4\pi f_c t + \phi)\right]
\end{aligned}$$

The low pass filter rejects the double frequency components and passes only the low pass components.

$$y(t) = \tfrac{1}{2} A_c m(t)\cos(\phi) + \tfrac{1}{2}\left[n_c(t)\cos(\phi) + n_s(t)\sin(\phi)\right]$$

the effect of a phase difference between the received carrier and a locally generated carrier at the receiver is a drop equal to $\cos^2(\phi)$ in the received signal power.

Phase-locked loop

The effect of a phase-locked loop is to generate phase of the received carrier at the receiver. If a phase-locked loop is employed, then $\phi = 0$ and the demodulator is called a coherent or synchronous demodulator.

In our analysis in this section, we assume that we are employing a coherent demodulator. With this assumption, we assume that $\phi = 0$

$$y(t) = \tfrac{1}{2}\left[A_c m(t) + n_c(t)\right]$$

Therefore, at the receiver output, the message signal and the noise components are additive and we are able to define a meaningful SNR. The message signal power is given by

$$P_o = \frac{A_c^2}{4} P_M$$

Power P_M is the content of the messagesignal

The noise power is given by

$$P_{n_0} = \frac{1}{4} P_{n_c} = \frac{1}{4} P_n$$

The power content of n(t) can be found by noting that it is the result of passing $n_W(t)$ through a filter with bandwidth B_c.Therefore, the power spectral density of n(t) is given by

$$S_n(f) = \begin{cases} \frac{N_0}{2} & |f - f_c| < W \\ 0 & otherwise \end{cases}$$

The noise power is

$$P_n = \int_{-\infty}^{\infty} S_n(f) df = \frac{N_0}{2} \times 4W = 2WN_0$$

Now we can find the output SNR as

$$\left(\frac{S}{N}\right)_0 = \frac{P_0}{P_{n_0}} = \frac{\frac{A_c^2}{4} P_M}{\frac{1}{4} 2WN_0} = \frac{A_c^2 P_M}{2WN_0}$$

In this case, the received signal power, given by

$$P_R = A_c{}^2 P_M / 2.$$

The output SNR for DSB-SC AM may be expressed as

$$\left(\frac{S}{N}\right)_{0_{DSB}} = \frac{P_R}{N_0 W}$$

which is identical to baseband SNR.

In DSB-SC AM, the output SNR is the same as the SNR for a baseband system. DSB-SC AM does not provide any SNR improvement over a simple baseband communication system.

NOISE IN SSB-SC SYSTEM:

Let SSB modulated signal is

$$u(t) = A_c m(t) \cos(2\pi f_c t) \mp A_c \hat{m}(t) \sin(2\pi f_c t)$$

Input to the demodulator

$$\begin{aligned} r(t) &= A_c m(t)\cos(2\pi f_c t) \mp A_c \hat{m}(t)\sin(2\pi f_c t) + n(t) \\ &= A_c m(t)\cos(2\pi f_c t) \mp A_c \hat{m}(t)\sin(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t) \\ &= [A_c m(t) + n_c(t)]\cos(2\pi f_c t) + [\mp A_c \hat{m}(t) - n_s(t)]\sin(2\pi f_c t) \end{aligned}$$

Assumption : Demodulation with an ideal phase reference.

Hence, the output of the lowpass filter is the in-phase component (with a coefficient of ½) of the precedingsignal.

Parallel to our discussion of DSB, we have

$$P_o = \frac{A_c^2}{4} P_M$$

$$P_{n_0} = \frac{1}{4} P_{n_c} = \frac{1}{4} P_n$$

$$P_n = \int_{-\infty}^{\infty} S_n(f)\,df = \frac{N_0}{2} \times 2W = WN_0$$

$$\rightarrow \left(\frac{S}{N}\right)_0 = \frac{P_0}{P_{n_0}} = \frac{A_c^2 P_M}{WN_0}$$

$$P_R = P_U = A_c^2 P_M$$

$$\left(\frac{S}{N}\right)_{0_{SSB}} = \frac{P_R}{N_0 W} = \left(\frac{S}{N}\right)_b$$

The signal-to-noise ratio in an SSB system is equivalent to that of a DSB system.

Noise in Conventional AM

DSB AM signal : $u(t) = A_c[1 + a m_n(t)]\cos(2\pi f_c t)$

Received signal at the input to the demodulator

$$\begin{aligned} r(t) &= A_c[1 + a m_n(t)]\cos(2\pi f_c t) + n(t) \\ &= A_c[1 + a m_n(t)]\cos(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t) \\ &= [A_c[1 + a m_n(t)] + n_c(t)]\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t) \end{aligned}$$

Where a is the modulation index

$m_n(t)$ is normalized so that its minimum value is -1

If a synchronous demodulator is employed, the situation is basically similar to the DSB case, except that we have $1 + a m_n(t)$ instead of $m(t)$.

$$y(t) = \tfrac{1}{2}\left[A_c a m_n(t) + n_c(t)\right]$$

Received signal power

$$P_R = \frac{A_c^2}{2}\left[1 + a^2 P_{M_n}\right]$$

- Assumed that the message process is zero mean.

Now we can derive the output SNR as

$$\left(\frac{S}{N}\right)_{0_{AM}} = \frac{\frac{1}{4}A_c^2 a^2 P_{M_n}}{\frac{1}{4}P_{n_c}} = \frac{A_c^2 a^2 P_{M_n}}{2N_0 W} = \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}} \frac{\frac{A_c^2}{2}\left[1 + a^2 P_{M_n}\right]}{N_0 W}$$

$$= \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}} \frac{P_R}{N_0 W} = \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}}\left(\frac{S}{N}\right)_b = \eta\left(\frac{S}{N}\right)_b$$

- η denotes the modulation efficiency
- Since $a^2 P_{M_n} < 1 + a^2 P_{M_n}$, the SNR in conventional AM is always smaller than the SNR in a baseband system.

- In practical applications, the modulation index a is in the range of 0.8-0.9.
- Power content of the normalized message process depends on the message source.
- Speech signals : Large dynamic range, P_M is about 0.1.
- The overall loss in SNR, when compared to a baseband system, is a factor of 0.075 or equivalent to a loss of 11 dB.

The reason for this loss is that a large part of the transmitter power is used to send the carrier component of the modulated signal and not the desired signal. To analyze the envelope-detector performance in the presence of noise, we must use certain approximations.

This is a result of the nonlinear structure of an envelope detector, which makes an exact analysis difficult

In this case, the demodulator detects the envelope of the received signal and the noise process.

The input to the envelope detector is

$$r(t) = \left[A_c[1 + a m_n(t)] + n_c(t)\right]\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

Therefore, the envelope of $r(t)$ is given by

$$V_r(t) = \sqrt{\left[A_c[1 + a m_n(t)] + n_c(t)\right]^2 + n_s^2(t)}$$

Now we assume that the signal component in *r (t)* is much stronger than the noise component. Then

$$P\left(n_c(t) << A_c[1+am_n(t)]\right) \approx 1$$

Therefore, we have a high probability that

$$V_r(t) \approx A_c[1+am_n(t)] + n_c(t)$$

After removing the DC component, we obtain

$$y(t) = A_c a m_n(t) + n_c(t)$$

which is basically the same as y*(t)* for the synchronous demodulation without the ½ coefficient.

This coefficient, of course, has no effect on the final SNR. So we conclude that, under the assumption of high SNR at the receiver input, the performance of synchronous and envelope demodulators is the same.

However, if the preceding assumption is not true, that is, if we assume that, at the receiver input, the noise power is much stronger than the signal power, Then

$$\begin{aligned}
V_r(t) &= \sqrt{\left[A_c[1+am_n(t)] + n_c(t)\right]^2 + n_s^2(t)} \\
&= \sqrt{A_c^2[1+am_n(t)]^2 + n_c^2(t) + n_s^2(t) + 2A_c n_c(t)[1+am_n(t)]} \\
&\xrightarrow{a} \sqrt{\left(n_c^2(t)+n_s^2(t)\right)\left[1+\frac{2A_c n_c(t)}{n_c^2(t)+n_s^2(t)}\left(1+am_n(t)\right)\right]} \\
&\xrightarrow{b} V_n(t)\left[1+\frac{A_c n_c(t)}{V_n^2(t)}\left(1+am_n(t)\right)\right] \\
&= V_n(t) + \frac{A_c n_c(t)}{V_n(t)}\left(1+am_n(t)\right)
\end{aligned}$$

(a) : $A_c^2[1+am_n(t)]^2$ is small compared with the other components

(b) : $\sqrt{n_c^2(t)+n_s^2(t)} = V_n(t)$;the envelope of the noise process

Use the approximation

$$\sqrt{1+\varepsilon} \approx 1+\tfrac{\varepsilon}{2}, \quad \textit{for small } \varepsilon, \text{ where } \varepsilon = \frac{2A_c n_c(t)}{n_c^2(t)+n_s^2(t)}\left(1+am_n(t)\right)$$

Then $$V_r(t) = V_n(t) + \frac{A_c n_c(t)}{V_n(t)}\left(1 + a m_n(t)\right)$$

We observe that, at the demodulator output, the signal and the noise components are no longer additive. In fact, the signal component is multiplied by noise and is no longer distinguishable. In this case, no meaningful SNR can be defined. We say that this system is operating below the threshold. The subject of threshold and its effect on the performance of a communication system will be covered in more detail when we discuss the noise performance in angle modulation.

Effect of threshold in angle modulation system:

FM threshold is usually defined as a Carrier-to-Noise ratio at which demodulated Signal-to-Noise ratio falls 1dB below the linear relationship . This is the effect produced in an FM receiver when noise limits the desired information signal. It occurs at about 10 dB, as earlier stated in 5 the introduction, which is at a point where the FM signal-to-Noise improvement is measured. Below the FM threshold point, the noise signal (whose amplitude and phase are randomly varying) may instantaneously have amplitude greater than that of the wanted signal. When this happens, the noise will produce a sudden change in the phase of the FM demodulator output. In an audio system, this sudden phase change makes a “click”. In video applications the term “click noise” is used to describe short horizontal black and white lines that appear randomly over a picture

An important aspect of analogue FM satellite systems is FM threshold effect. In FM systems where the signal level is well above noise received carrier-to-noise ratio and demodulated signal-to-noise ratio are related by:

$$S/N = 3\beta^2 C/N \qquad \text{Eqn 9}$$

where S/N = signal-to-noise ratio at output of FM demodulator
β = FM deviation ratio or modulation index = ($\Delta f/B$)
C/N = carrier-to-noise ratio at input of FM demodulator
Δf = peak deviation
B = basebandwidth of signal being modulated

The expression however does not apply when the carrier-to-noise ratio decreases below a certain point. Below this critical point the signal-to-noise ratio decreases significantly. This is known as the FM threshold effect (FM threshold is usually defined as the carrier-to-noise ratio at which the demodulated signal-to-noise ratio fall 1 dB below the linear relationship given in Eqn 9. It generally is considered to occur at about 10 dB).

Below the FM threshold point the noise signal (whose amplitude and phase are randomly varying), may instantaneously have an amplitude greater than that of the wanted signal. When this happens the noise will produce a sudden change in the phase of the FM demodulator output. In an audio system this sudden phase change makes a "click". In video applications the term "click noise" is used to describe short horizontal black and white lines that appear randomly over a picture, because satellite communications systems are power limited they usually operate with only a small design margin above the FM threshold point (perhaps a few dB). Because of this circuit designers have tried to devise techniques to delay the onset of the FM threshold effect. These devices are generally known as FM threshold extension demodulators. Techniques such as FM feedback, phase locked loops and frequency locked loops are used to achieve this effect. By such techniques the onset of FM threshold effects can be delayed till the C/N ratio is around 7 dB.

Noise in Angle Modulated Systems

Like AM, noise performance of angle modulated systems is characterized by parameter γ

$$\gamma_{FM} = \frac{3}{2}\beta^2$$

If it is compared with AM

$$\frac{\gamma_{FM}}{\gamma_{AM}} = \frac{1}{2}\left(\frac{\omega_{FM}}{\omega_{AM}}\right)^2$$

Note: if bandwidth ratio is increased by a factor 2.Then $\frac{\gamma_{FM}}{\gamma_{AM}}$ increases by a factor 4

This exchange of bandwidth and noise performance is an important feature of FM

$$\text{Figure of merit } (\gamma) = \frac{SNR_O}{SNR_C}$$

CW- Modulation System	SNR_O	SNR_C	Figure of merit	Figure of merit (single tone)
DSB-SC	$\frac{C^2 A_c^2 P}{2WN_0}$	$\frac{C^2 A_c^2 P}{2WN_0}$	1	1
SSB	$\frac{C^2 A_c^2 P}{4WN_0}$	$\frac{C^2 A_c^2 P}{4WN_0}$	1	1
AM	$\frac{A_c^2 k_a^2 P}{2WN_0}$	$\frac{A_c^2 (1+k_a^2 P)}{2WN_0}$	$\approx \frac{k_a^2 P}{1+k_a^2 P} < 1$	$\frac{\mu^2}{2+\mu^2}$
FM	$\frac{3A_c^2 k_f^2 P}{2N_0 W^3}$	$\frac{A_c^2}{2WN_0}$	$\frac{3k_f^2 P}{W^2}$	$\frac{3}{2}\beta^2$

P is the average power of the message signal.

C^2 is a constant that ensures that the ration is dimensionless.

W is the message bandwidth.

A_c is the amplitude of the carrier signal.

k_a is the amplitude sensitivity of the modulator.

$\mu = k_a A_m$ and A_m is the amplitude sinusoidal wave

$\beta = \frac{\Delta f}{W}$ is the modulation index.

k_f is the frequency sensitivity of the modulator.

Δf is the frequency deviation.

UNIT-5

ANALOG PULSE MODULATION SCHEMES

- Pulse amplitude modulation (PAM) & demodulation
- Transmission bandwidth
- Pulse-Time Modulation,
- Pulse Duration
- Pulse Position modulations and demodulation schemes
- Multiplexing Techniques, FDM,
- TDM.
- **Information Theory**
- Introduction to information theory
- Entropy
- Mutual information
- Channel capacity theorem
- Shannon-Fano encoding algorithm
- Illustrative Problems.

Introduction:

Pulse Modulation

- Carrier is a train of pulses
- Example: Pulse Amplitude Modulation (PAM), Pulse width modulation (PWM) , Pulse Position Modulation (PPM)

Types of Pulse Modulation:

- The immediate result of sampling is a pulse-amplitude modulation (PAM) signal
- PAM is an analog scheme in which the amplitude of the pulse is proportional to the amplitude of the signal at the instant of sampling
- Another analog pulse-forming technique is known as **pulse-duration modulation (PDM).** This is also known as **pulse-width modulation (PWM)**
- **Pulse-position modulation** is closely related to PDM

Pulse Amplitude Modulation:

In PAM, amplitude of pulses is varied in accordance with instantaneous value of modulating signal.

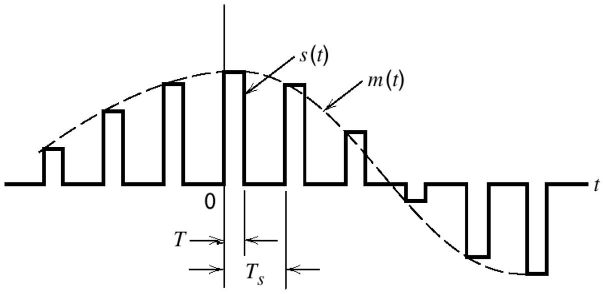


Fig.5.1. PAM

PAM Generation:

The carrier is in the form of narrow pulses having frequency fc. The uniform sampling takes place in multiplier to generate PAM signal. Samples are placed Ts sec away from each other.

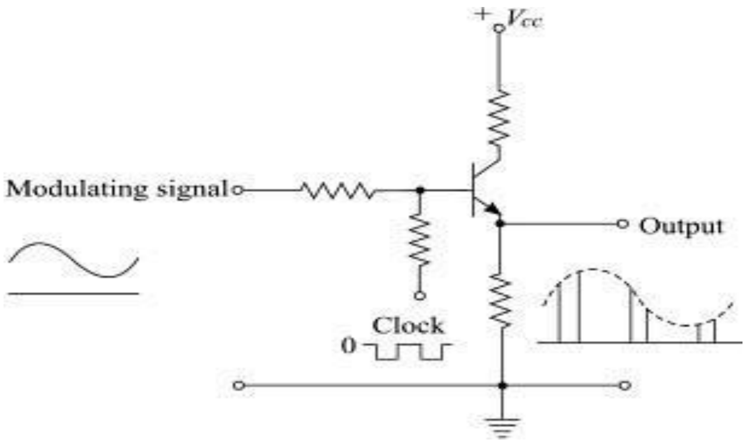


Fig.5.2. PAM Modulator

- The circuit is simple emitter follower.
- In the absence of the clock signal, the output follows input.
- The modulating signal is applied as the input signal.
- Another input to the base of the transistor is the clock signal.
- The frequency of the clock signal is made equal to the desired carrier pulse train frequency.
- The amplitude of the clock signal is chosen the high level is at ground level(0v) and low level at some negative voltage sufficient to bring the transistor in cutoff region.
- When clock is high, circuit operates as emitter follower and the output follows in the input modulating signal.
- When clock signal is low, transistor is cutoff and output is zero.
- Thus the output is the desired PAM signal.

PAM Demodulator:

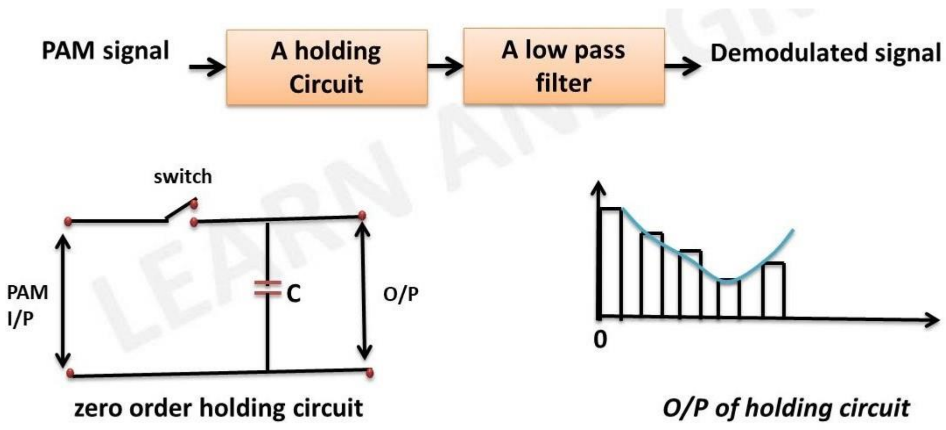
- 

Fig.5.3. PAM Demodulator

The PAM demodulator circuit which is just an envelope detector followed by a second order op-amp low pass filter (to have good filtering characteristics) is as shown

Pulse Width Modulation:

- In this type, the amplitude is maintained constant but the width of each pulse is varied in accordance with instantaneous value of the analog signal.

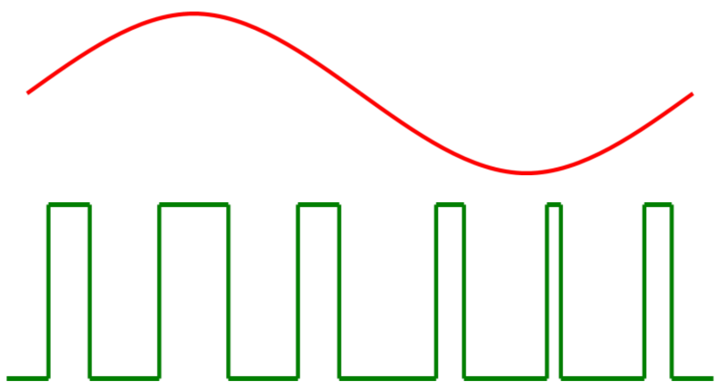


Fig.5.4. PWM Waveforms

- In PWM information is contained in width variation. This is similar to FM.
- In pulse width modulation (PWM), the width of each pulse is made directly proportional to the amplitude of the information signal.

Pulse Position Modulation:

- In this type, the sampled waveform has fixed amplitude and width whereas the position of each pulse is varied as per instantaneous value of the analog signal.
- PPM signal is further modification of a PWM signal.

PPM & PWM Modulator:

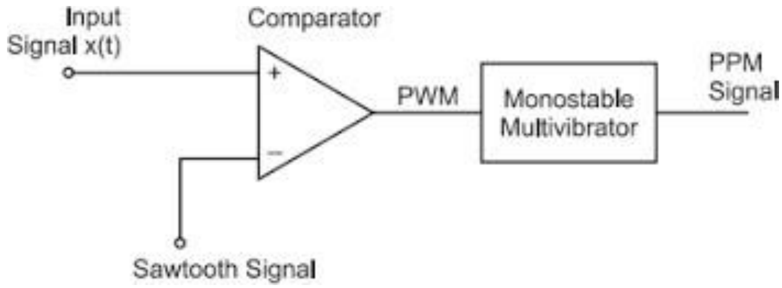


Fig.5.5. PWM & PPM Modulator

- The PPM signal can be generated from PWM signal.
- The PWM pulses obtained at the comparator output are applied to a mono stable multi vibrator which is negative edge triggered.

- Hence for each trailing edge of PWM signal, the monostable output goes high. It remains high for a fixed time decided by its RC components.
- Thus as the trailing edges of the PWM signal keeps shifting in proportion with the modulating signal, the PPM pulses also keep shifting.
- Therefore all the PPM pulses have the same amplitude and width. The information is conveyed via changing position of pulses.

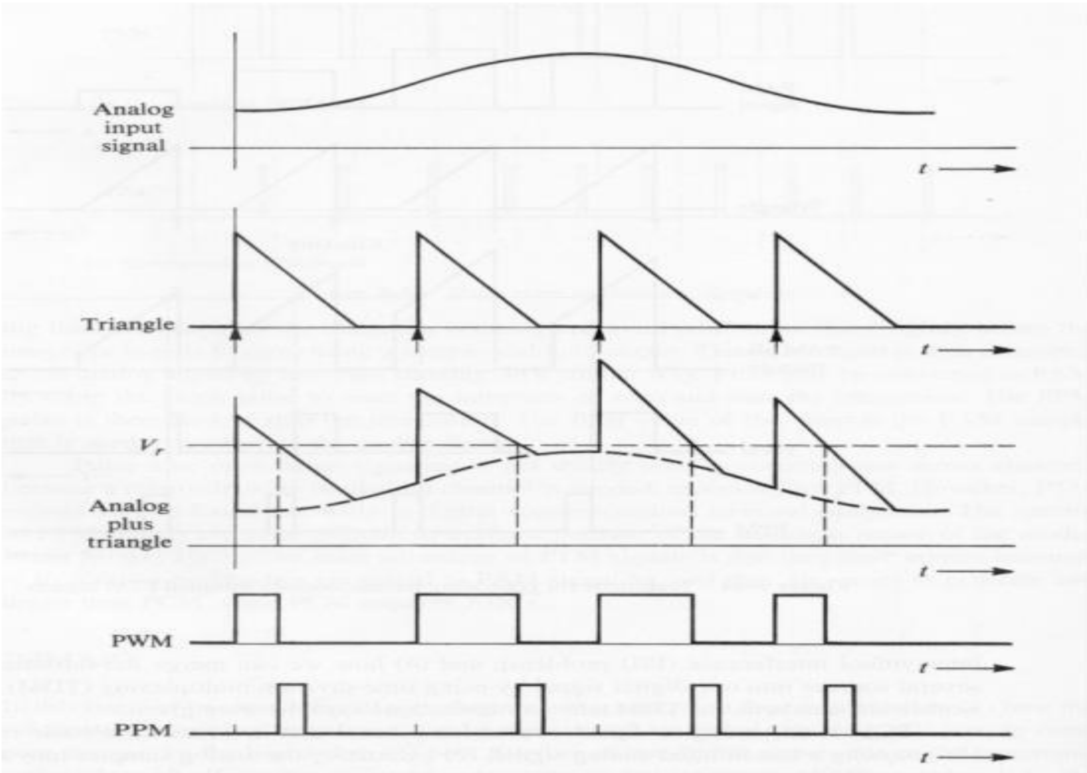


Fig.5.6. PWM & PPM Modulation waveforms

PWM Demodulator:

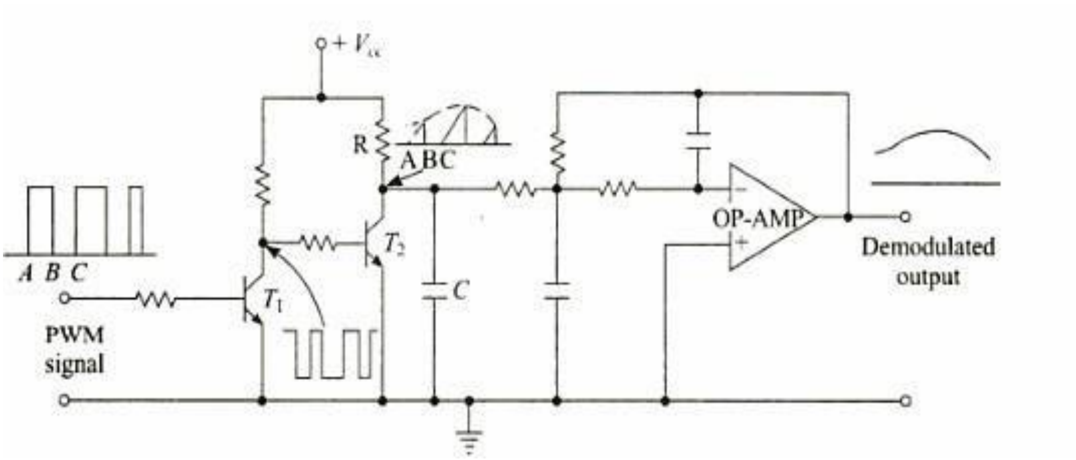


Fig.5.7. PWM Demodulator

- Transistor T1 works as an inverter.
- During time interval A-B when the PWM signal is high the input to transistor T2 is low.
- Therefore, during this time interval T2 is cut-off and capacitor C is charged through an R-C combination.
- During time interval B-C when PWM signal is low, the input to transistor T2 is high, and it gets saturated.
- The capacitor C discharges rapidly through T2.The collector voltage of T2 during B-C is low.
- Thus, the waveform at the collector of T2is similar to saw-tooth waveform whose envelope is the modulating signal.
- Passing it through 2^{nd} order op-amp Low Pass Filter, gives demodulated signal.

PPM Demodulator:

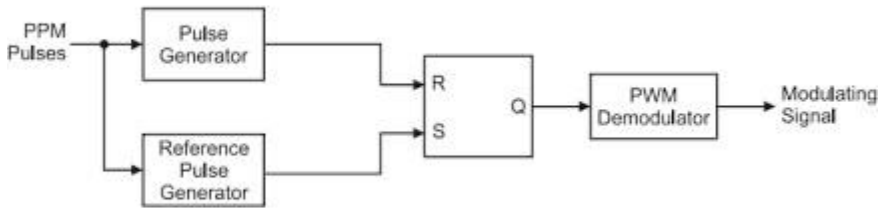


Fig.5.8. PPM Demodulator

- The gaps between the pulses of a PPM signal contain the information regarding the modulating signal.
- During gap A-B between the pulses the transistor is cut-off and the capacitor C gets charged through R-C combination.
- During the pulse duration B-C the capacitor discharges through transistor and the collector voltage becomes low.
- Thus, waveform across collector is saw-tooth waveform whose envelope is the modulating signal.
- Passing it through 2^{nd} order op-amp Low Pass Filter, gives demodulated signal.

MULTIPLEXING TECHNIQUES

A multiplexing technique by which multiple data signals can be transmitted over a common communication channel in different time slots is known as Time Division Multiplexing (TDM).It allows the division of the overall time domain into various fixed length time slots. A single frame is said to be transmitted when it's all signal components gets transmitted over the channel. Multiplexing allows the transmission of several signals over a common channel. However, one may need to differentiate between the various signal for proper data transmission. So, in time division multiplexing, the complete signal gets transmitted by occupying different time slots. The name itself is indicating here that basically time division is performed in order to multiplex multiple data signals. Let us have a look at the figure below in order to have a better understanding of the TDM process.

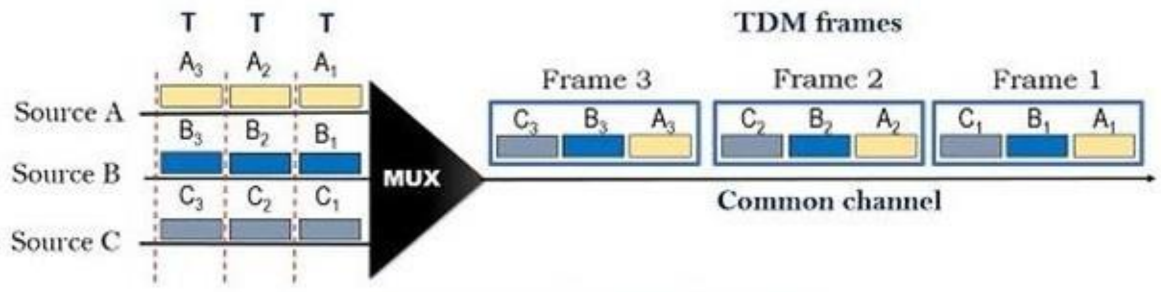


Fig.5.9. TDM Working Principle

As we can see that source A, B and C wants to transmit data through a common medium. Thus, the signal from the 3 sources, is divided into multiple frames each having their fixed time slot. Here, 3 units from each source are taken into consideration, that jointly form the actual signal. A frame is transmitted at a time that is composed of one unit of each source. As these units are entirely different from each other thus the chances of unnecessary signal mixing can be eliminated. When a frame gets transmitted over the particular time slot, the next frame uses the same channel to get transmitted and the process is further repeated until the completion of the transmission. Here, we have taken the example of 3 different sources, but one can perform multiplexing of n source signals. It is noteworthy here that units of a single source must be equivalent to the total number of source signals to be transmitted. Both analog and digital signals can be multiplexed using time division multiplexing, but its processing technique allows the multiplexing of digital signals conveniently rather than analog one.

TIME DIVISION MUTIPLEXING

The figure below shows the block diagram of a TDM system employing both transmitter and receiver section

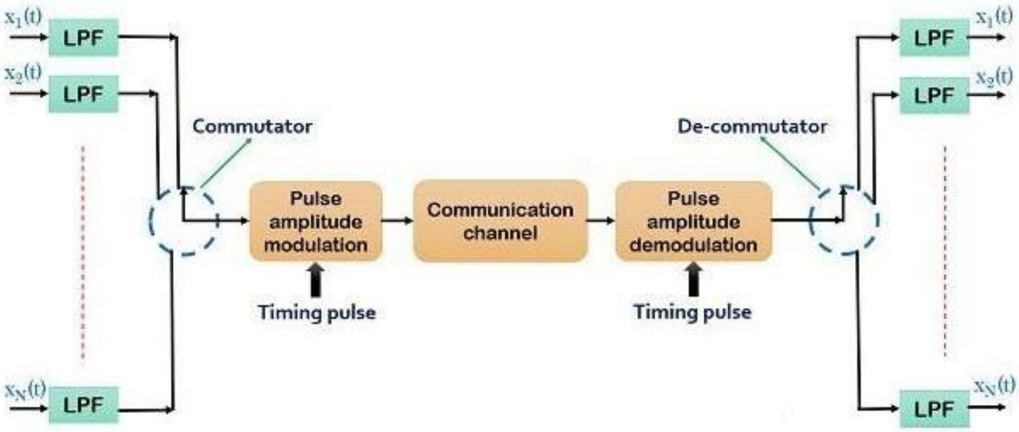


Fig.5.10. Block Diagram of TDM

The technique efficiently utilizes the complete channel for data transmission hence sometimes known as PAM/TDM. This is so because a TDM system uses a pulse amplitude modulation. In this modulation technique, each pulse holds some short time duration allowing maximal channel usage.Here at the beginning, the system consists of multiple LPF depending on the number of data inputs. These low pass filters are basically anti-aliasing filters that eliminate the aliasing of the data input signal.

The output of the LPF is then fed to the commutator. As per the rotation of the commutator the samples of the data inputs are collected by it. Here, f_s is the rate of rotation of the commutator, thus denotes the sampling frequency of the system. Suppose we have n data inputs, then one after the other, according to the rotation, these data inputs after getting multiplexed transmitted over the common channel.Now, at the receiver end, a de-commutator is placed that is synchronized with the commutator at the transmitting end. This de-commutator separates the time division multiplexed signal at the receiving end. The commutator and de-commutator must have same rotational speed so as to have accurate demultiplexing of the signal at the receiving end. According to the rotation performed by the de-commutator, the samples are collected by the LPF and the original data input is recovered at the receiver.

In this way a TDM works.

Let f_m be the maximum signal frequency and f_s is the sampling frequency then

$$f_s \geq 2f_m$$

Thus, the time duration in between successive sample is given as, $T_s = \frac{1}{f_s}$

Rewriting in terms of f_m $T_s \leq \frac{1}{2f_m}$

Now, as we have considered that there are N input channels, then one sample is collected from each of the N samples.

Hence, each interval will provide us with N samples and the spacing between the two is given as

$$\frac{T_s}{N}$$

We know pulse frequency is basically the number of pulses per second and is given by

$$\text{Pulse frequency} = \frac{1}{Spacing\ between\ 2\ samples}$$

$$= \frac{1}{\frac{T_s}{N}}$$

$$= \frac{N}{T_s}$$

$$= \frac{N}{\frac{1}{f_s}}$$

$$= Nf_s$$

For a TDM signal pulse per second is the signaling rate denoted as ‘r’.

Thus,

$$r = N\,f_s$$

FREQUENCY DIVISION MUTIPLEXING

The operation of frequency division multiplexing (FDM) is based on sharing the available bandwidth of a communication channel among the signals to be transmitted. This means that many signals are transmitted simultaneously with each signal occupying a different frequency slot within a common bandwidth. Each signal to be transmitted modulates a different carrier. The modulation can be AM,SSB, FM or PM .The modulated signals are then added together to form a composite signal which is transmitted over a single channel.The spectrum of composite FDM signal has been shown in fig.1.

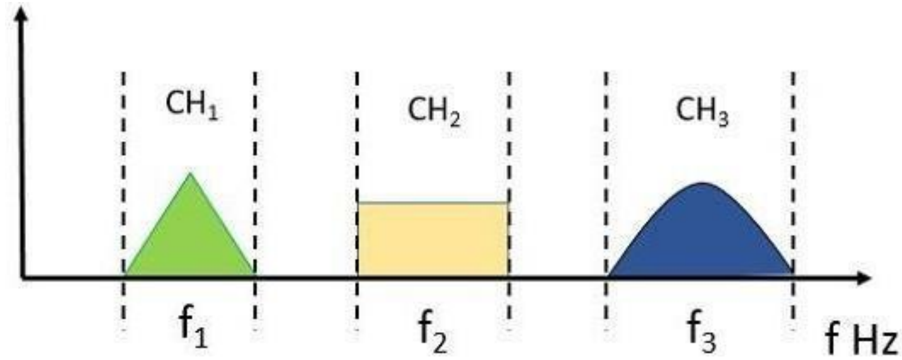


Fig.5.11: Spectrum of FDM Signal

Generally, the FDM systems are used for multiplexing the analog signals.

FDM Transmitter

Fig. shows the block diagram of an FDM transmitter.

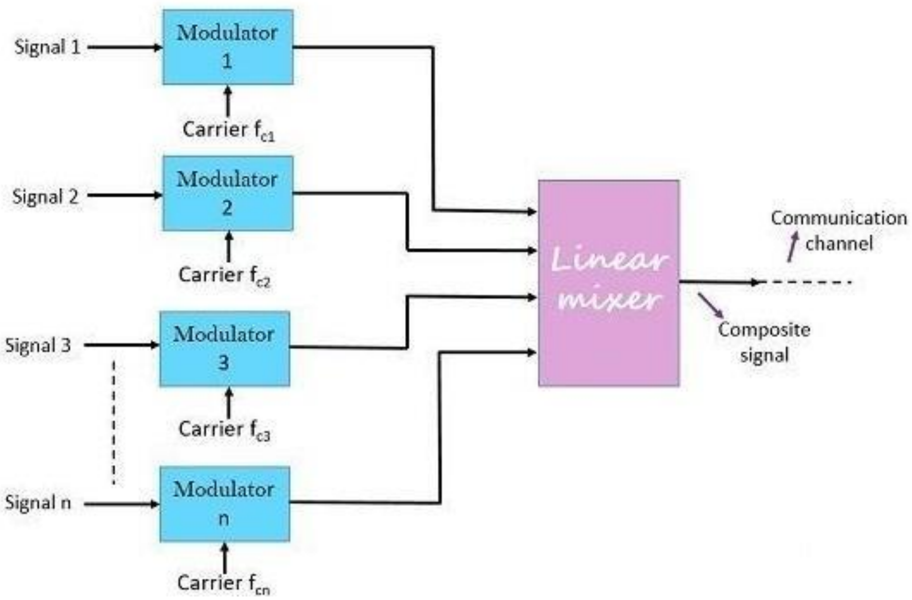


Fig. 5.12: FDM Transmitter

The signals which are to be multiplexed will each modulate a separate carrier .The type of modulation can be AM, SSB, FM or PM .The modulated signals are then added together to form a complex signal which is transmitted over a single channel .

Working Operation of the FDM Transmitter

Each signal modulates a separate carrier. The modulator outputs will contain the sidebands of the corresponding signals.The modulator outputs are added together in a linear mixer or adder. The linear mixer is different from the normal mixers. Here the sum and difference frequency components are not produced. But only the algebraic addition of the modulated outputs will take place.Different signals are thus added together i the time domain but they have a separate identity in the frequency domain. This is as shown in fig.2.The composite signal at the output of mixer is transmitted over the single communication channel as shown in fig.2. This signal can be used to modulate a radio transmitter if the FDM signal is to be transmitted through air.

FDM Receiver

The block diagram of an FDM receiver is shown in fig.

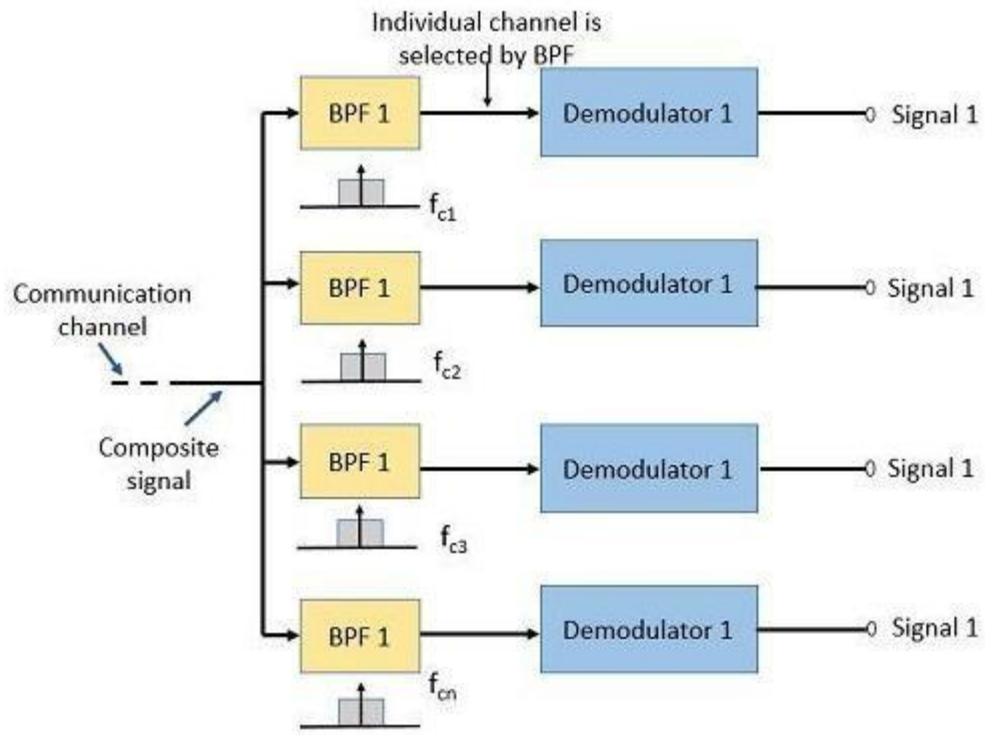


Fig.5.13: FDM Receiver

The composite signal is applied to a group of bandpass filters (BPF) .Each BPF has a center frequency corresponding to one of the carriers. The BPFs have an adequate bandwidth to pass all the channel information without any distortion .Each filter will pass only its channel and rejects all the other channels .The channel demodulator then removes the carrier and recovers the original signal back .

INTRODUCTION TO INFORMATION THEORY

INFORMATION:

Information is the source of a communication system, whether it is analog or digital. **Information theory** is a mathematical approach to the study of coding of information along with the quantification, storage, and communication of information.

Conditions of Occurrence of Events

If we consider an event, there are three conditions of occurrence.

- If the event has not occurred, there is a condition of **uncertainty**.
- If the event has just occurred, there is a condition of **surprise**.
- If the event has occurred, a time back, there is a condition of having some **information**.

These three events occur at different times. The difference in these conditions help us gain knowledge on the probabilities of the occurrence of events.

Entropy;

When we observe the possibilities of the occurrence of an event, how surprising or uncertain it would be, it means that we are trying to have an idea on the average content of the information from the source of the event.

Entropy can be defined as a measure of the average information content per source symbol. **Claude Shannon**, the “father of the Information Theory”, provided a formula for it as –

H=−∑ipilogbpiH=−∑ipilogbpi

Where $\mathbf{p_i}$ is the probability of the occurrence of character number **i** from a given stream of characters and **b** is the base of the algorithm used. Hence, this is also called as **Shannon’s Entropy**.

The amount of uncertainty remaining about the channel input after observing the channel output, is called as **Conditional Entropy**. It is denoted by H(x|y)H(x|y)

Properties of Entropy:

1. Entropy is always non negative i.e $H(x) \geq 0$.

2. Entropy is zero when probability of all symbols is zero except probability one symbol is one.

3. Entropy is maximum when probability occurrence of all symbols is equal

i.e $H(x) = log_2 M$

Mutual Information

Let us consider a channel whose output is **Y** and input is **X**

Let the entropy for prior uncertainty be **X = H**xx
Thisisassumedbeforetheinputisapplied Thisisassumedbeforetheinputisapplied

To know about the uncertainty of the output, after the input is applied, let us consider Conditional Entropy, given that $\mathbf{Y = y_k}$

Now, considering both the uncertainty conditions beforeandafterapplyingtheinputsbeforeandafterapplyingtheinputs, we come to know that the difference, i.e. H(x)−H(x|y)H(x)−H(x|y) must represent the uncertainty about the channel input that is resolved by observing the channel output.

This is called as the **Mutual Information** of the channel.

Denoting the Mutual Information as I(x;y)I(x;y), we can write the whole thing in an equation, as follows

$$I(x;y)=H(x)-H(x|y) \quad I(x;y)=H(x)-H(x|y)$$

Hence, this is the equational representation of Mutual Information.

Properties of Mutual information

- I(X; Y) of a channel is equal to difference between initial uncertainty and final uncertainty.
- I(X;Y) = Initial uncertainty – final uncertainty.
- I(X;Y) = H(X) - H(X/Y) bits/symbol
  Where, H(X) - entropy of the source and
  H(X/Y) - Conditional Entropy.

Properties of mutual information:

1. I(X;Y) = I(Y;X)
2. I(X;Y)>=0
3. I(X;Y) = H(X) - H(X/Y)
4. I(X;Y)) = H(X)+H(Y)-H(X,Y).

Channel Capacity

We have so far discussed mutual information. The maximum average mutual information, in an instant of a signaling interval, when transmitted by a discrete memoryless channel, the probabilities of the rate of maximum reliable transmission of data, can be understood as the **channel capacity**.

$$C = W\ log2\left[1 + \frac{S}{N}\right] bits/sec$$

Where, W= Channel bandwidth

S = Average signal power

N = Average noise power

It is denoted by **C** and is measured in **bits per channel** use.

The Shannon-Fano Encoding Algorithm

1. Calculate the frequency of each of the symbols in the list.
2. Sort the list in (decreasing) order of frequencies.
3. Divide the list into two half's, with the total frequency counts of each half being as close as possible to each other.
4. The right half is assigned a code of 1 and the left half with a code of 0.
5. Recursively apply steps 3 and 4 to each of the halves, until each symbol has become a corresponding code leaf on the tree. That is, treat each split as a list and apply splitting and code assigning till you are left with lists of single elements.
6. Generate code word for each symbol

   Let us assume the source alphabet S={X1,X2,X3,Ö,Xn} and Associated probability P={P1,P2,P3 ,Ö,Pn}. The steps to encode data using Shannon-Fano coding algorithm is as follows: Order the source letter into a sequence according to the probability of occurrence in non-increasing order i.e. decreasing order.

SHANNON-FANO CODING

Ques:- Apply Shannon-Fano Coding for following message

[X] : [x1 x2 x3 x4 x5 x6]

[P] : [0.30 0.25 0.15 0.12 0.08 0.10]

Take M=2

[X]	[P]	Stage 1	Stage 2	Stage 3	Codeword	Length
x_1	0.30	0	0		00	2
x_2	0.25	0	1		01	2
x_3	0.15	1	0	0	100	3
x_4	0.12	1	0	1	101	3
x_6	0.10	1	1	0	110	3
x_5	0.08	1	1	1	111	3

$$H(x) = \sum_{i=1}^{6} P_i n_i$$

$$= (0.30)\log_2\frac{1}{(0.30)} + (0.25)\log_2\frac{1}{(0.25)} + (0.15)\log_2\frac{1}{(0.15)} + (0.12)\log_2\frac{1}{(0.12)} + (0.10)\log_2\frac{1}{(0.10)} + (0.08)\log_2\frac{1}{(0.08)}$$

$$= 2.418 \text{ Bits}$$

$$L_{avg} = \sum_{i=1}^{6} P_i \text{ (length)}$$

$$= (0.30\times2) + (0.25\times2) + (0.15\times3) + (0.12\times3) + (0.10\times3) + (0.08\times3)$$

$$= 2.45 \text{ Bits}$$

(η) Efficiency $= \dfrac{H(x)}{L_{avg}}$

$$= \frac{2.418}{2.45} = 0.9869.$$

98%

Redundancy

$R = 1 - \eta$

$= 1 - 0.9869$

$= 0.0131$