



SIDDHARTH GROUP OF INSTITUTIONS :: PUTTUR
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QUESTION BANK (DESCRIPTIVE)

Subject with Code : Mathematics-I (18HS0830)

Course & Branch: B.Tech – ALL

Year & Sem: I-I

Regulation: R18

UNIT –I

1. a) Find the Rank of $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$. [2M]
- b) Reduce the matrix $A = \begin{bmatrix} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 3 & -3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$ into Echelon form and find its rank? [2M]
- c) Define Symmetric & Skew-symmetric matrices. [2M]
- d) If $A = \begin{bmatrix} 3 & a & b \\ -2 & 2 & 4 \\ 7 & 4 & 5 \end{bmatrix}$ is symmetric, then find a, b values? [2M]
- e) Find the Eigen values of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. [2M]
2. a) Define the rank of the Matrix. [2M]
- b) Find whether the following equations are consistent if so solve them $x + y + 2z = 4$; $2x - y + 3z = 9$; $3x - y - z = 2$. [8M]
3. a) Show that the matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is a orthogonal matrix. [5M]
- b) Express the matrix $A = \begin{bmatrix} 3 & -2 & -6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrix. [5M]
4. a) Find the rank of the matrix $A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ [5M]
- b) Determine the Eigen values of A^{-1} where $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. [5M]
5. a) Show that $A = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$ is orthogonal. [5M]
- b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ [5M]
6. a) State Cayley-Hamilton theorem [2M]

- b) Show that the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & -2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation. [8M]
7. Find the Eigen values and corresponding Eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. [10M]
8. Diagonalise the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ and hence find A^4 . [10M]
9. Reduce the Quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ into sum of squares form by Orthogonal transformation. [10M]
10. Reduce the Quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy + 2xz - 2yz$ into the canonical form by Orthogonal transformation. [10M]

UNIT -II

1. a) Evaluate the improper integral $\int_1^{\infty} \frac{1}{x^4} dx$. [2M]
 b) Define gamma and beta function. [2M]
 c) Prove that $\Gamma(1)=1$. [2M]
 d) State Rolle's theorem. [2M]
 e) State Lagrange's mean value theorem. [2M]
2. a) Find the surface area generated by the revolution of an arc of (catenary) curve $y = c \cdot \cosh \frac{x}{c}$ from $x=0$ to $x=c$ about the x-axis. [5M]
 b) Find the volume of solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the major axis. [5M]
3. a) Find the surface area of the sphere of radius 'a'. [5M]
 b) Find the volume of the reel-shaped solid formed by the revolution about the y- axis, of the part of the parabola $y^2 = 4ax$ cut off by the latus- rectum. [5M]
4. a) State and verify the Rolle's theorem then $f(x) = \log \left[\frac{x^2 + ab}{x(a+b)} \right]$ in $[a, b]$. [5M]
 b) Verify lagrange's mean value theorem for $f(x) = x^3 - x^2 - 5x + 3$ in $[0, 4]$. [5M]
5. a) verify Cauchy's mean value theorem for the function $\sin x$ and $\cos x$ in the interval $\left[0, \frac{\pi}{2}\right]$. [5M]
 b) Express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$ assigning Taylor's series. [5M]
- 6.a) Calculate the approximate value of $\sqrt{10}$ correct to 4 decimal places using Taylor's theorem. [5M]
 b) Expand $\log_e x$ in power of $(x - 1)$ and hence evaluate $\log 1.1$ correct to 4 decimal places using Taylor's theorem. [5M]

7. a) Using Maclaurin's series expand $\tan x$ up to the fifth power of x and hence find the series for $\log(\sec x)$. [5M]
 b) Evaluate $\int_0^1 x^2 (\log \frac{1}{x})^3$. [5M]
8. a) Prove that $\int_0^1 \frac{x}{\sqrt{1-x^5}} dx = \frac{1}{5} B(\frac{2}{5}, \frac{1}{2})$. [5M]
 b) Prove that $\int_0^1 \frac{x}{\sqrt{1-x^2}} dx = \frac{1}{2} \beta(1, \frac{1}{2})$. [5M]
9. a) Prove that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. [5M]
 b) Evaluate $\int_0^\infty \sqrt{x} e^{-x^2} dx$ [5M]
10. a) Prove that $\beta(m, n) = 2 \int_0^{\frac{\pi}{2}} \sin^{2m-1} \theta \cdot \cos^{2n-1} \theta d\theta$. [5M]
 b) Prove that $\int_0^1 (\log \frac{1}{x})^{n-1} dx = \tau(n)$. [5M]

UNIT -III

- 1) a) Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$. [2M]
 b) Define Total differential Coefficient. [2M]
 c) Find the stationary points of $f(x, y) = x^3 + y^3 - 3axy$. [2M]
 d) Define CURL of a Vector. [2M]
 e) If $\vec{f} = xy^2\vec{i} + 2x^2yz\vec{j} - 3yz^2\vec{k}$ find $\text{div. } \vec{f}$ at $(1, -1, 1)$. [2M]
- 2) a) Discuss the continuity of the function $f(x, y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & (x, y) \neq (0,0) \\ 0 & \text{at } (0,0) \end{cases}$ [5M]
 b) If $U = \frac{1}{\sqrt{x^2+y^2+z^2}}$; $x^2+y^2+z^2 \neq 0$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$. [5M]
- 3) a) If $u = \tan^{-1} \left[\frac{2xy}{x^2-y^2} \right]$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. [5M]
 b) If $U = \log(x^3+y^3+z^3-3xyz)$ prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 U = \frac{-9}{(x+y+z)^2}$. [5M]
- 4) a) Find $\frac{du}{dt}$ as a total derivative; if $u = x^2y^3$ where $x = \log t$ and $y = e^t$ [5M]
 b) If $z = xy^2 + x^2y$; where $x = at^2, y = 2at$, find $\frac{dz}{dt}$ as a total derivative. [5M]
- 5) a) If $u = \sin^{-1}(x-y)$, where $x = 3t, y = 4t^3$, then show that $\frac{du}{dt} = \frac{3}{\sqrt{1-t^2}}$. [5M]
 b) If $u = x^2+y^2+z^2$ and $x = e^{2t}, y = e^{2t}\cos 3t, z = e^{2t}\sin 3t$, find $\frac{du}{dt} = ?$ [5M]
- 6) a) Examine the function for extreme values $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$; $(x>0, y>0)$ [5M]
 b) Find the stationary points of $u(x, y) = \sin x \cdot \sin y \cdot \sin(x+y)$ where $0 < x < \pi, 0 < y < \pi$ and find the maximum of u . [5M]

- 7) a) Find the shortest distance from origin to the surface $xyz^2 = 2$. [5M]
 b) Find the minimum value of $x^2 + y^2 + z^2$ given $x + y + z = 3a$. [5M]
- 8) a) Find a point on the plane $3x + 2y + z - 12 = 0$, which is nearest to the origin. [5M]
 b) Find the shortest and longest distance from the point $(3, 1, -1)$ to the sphere $x^2 + y^2 + z^2 = 4$. [5M]
- 9) a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P = (1, 2, 3)$ in the direction of the line PQ where $Q = (5, 0, 4)$. [5M]
 b) Find the directional derivative of $f(x, y, z) = 2xy + z^2$ at $(1, -1, 3)$ in the direction of $\vec{i} + 2\vec{j} + 3\vec{k}$. [5M]
- 10) a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. [5M]
 b) Find $\text{curl } \vec{f}$ where $\vec{f} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. [5M]

UNIT -IV

1. a) Define Convergent and Divergent of a sequence [2M]
 b) Examine the sequence $a_n = 2^n$ for convergence. [2M]
 c) Define Sequence and Series. [2M]
 d) Test for convergence the series $\sum \frac{n^3}{3^n}$. [2M]
 e) Define Power Series. [2M]
2. Examine the following sequences for convergence:
 i) $a_n = \frac{n^2 - 2n}{3n^2 + n}$ ii) $a_n = 3 + (-1)^n$. [10M]
3. Show that the series $1 + r + r^2 + r^3 + \dots \infty$
 i) Converges if $|r| < 1$ ii) Diverges if $r \geq 1$ and iii) Oscillates if $r \leq -1$. [10M]
4. Show that the p - series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots \infty$
 i) Converges for $p > 1$ ii) Diverges for $p \leq 1$ [10M]
5. Test for convergence the series
 i) $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots \infty$ ii) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$. [10M]
6. Discuss the convergence of the series i) $\sum \left(\frac{n!}{(n^n)^2} \right)$ ii) $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \dots \infty$ [10M]
7. a) Test for convergence the series $\sum \left(1 + \frac{1}{\sqrt{n}} \right)^{-n^{\frac{3}{2}}}$. [5M]
 b) Discuss the nature of the series $\sum \frac{(n+1)^n x^n}{x^{n+1}}$. [5M]
8. State the value of x, for which the following series converge:
 i) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots \infty$. ii) $\frac{1}{1-x} + \frac{1}{2(1-x)^2} + \frac{1}{3(1-x)^3} + \dots \infty$. [10M]
9. a) Show that the exponential series
 $1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots \infty$ is convergent for all values of x. [5M]

- b) For what values of x the series $x + \frac{2!}{2^2}x^2 + \frac{3!}{3^3}x^3 + \dots + \frac{n!}{n^n}x^n + \dots$ is convergent. [5M]
10. a) Discuss the convergence of the series $\sum \frac{1}{\sqrt{n}} \tan \frac{1}{n}$. [5M]
- b) Test for convergence of the series $\sum \log \left(1 + \frac{1}{n}\right)$. [5M]

UNIT -V

1. a) Find Fourier coefficient b_n when $f(x) = e^x$ in $[-\pi, \pi]$. [2M]
 b) If $f(x) = |\sin x|$ in $-\pi < x < \pi$ then find Fourier coefficient a_0 . [2M]
 c) Find the half-range sine series for $f(x) = 1$ in $(0, \pi)$ [2M]
 d) Obtain the Fourier series for $f(x) = \pi x$ in $0 \leq x \leq 2$ [2M]
 e) Find Fourier constant a_0 for $f(x) = 1 - x^2$ in $[-1, 1]$. [2M]
2. Find a Fourier series to represent the function $f(x) = e^x$ for $-\pi < x < \pi$. and hence derive a series for $\frac{\pi}{\sinh \pi}$. [10M]
3. a) Obtain the Fourier series expansion of $f(x) = (\pi - x)^2$ in $0 < x < 2\pi$ and deduce the value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$. [5M]
 b) Find the Fourier series for the function $f(x) = x$; in $-\pi < x < \pi$. [5M]
4. Find the Fourier series to represent the function $f(x) = x^2$ for $-\pi < x < \pi$ and hence show that
 (i) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$. (ii) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$.
 (iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$ [10M]
5. a) If $f(x) = |\sin x|$, expand $f(x)$ as a Fourier series in the interval $(-\pi, \pi)$ [5M]
 b) Find the half range cosine series for $f(x) = x$ in the interval $0 \leq x \leq \pi$. [5M]
6. Expand the function $f(x) = |x|$ in $-\pi < x < \pi$ as a Fourier series and Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$ [10M]
7. Find the half range sine series for $f(x) = x(\pi - x)$ in the interval $0 \leq x \leq \pi$ and Deduce that $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots = \frac{\pi^3}{32}$. [10M]
8. a) Expand $f(x) = e^{-x}$ as a Fourier series in the interval $(-1, 1)$. [5M]
 b) Expand $f(x) = |x|$ as a Fourier series in the interval $(-2, 2)$. [5M]
9. a) Find the half range sine series expansion of $f(x) = x^2$ when $0 < x < 4$. [5M]
 b) Find the half range cosine series expansion of $f(x) = x(2 - x)$ in $0 \leq x \leq 2$. [5M]
10. Find half range Fourier cosine series of $f(x) = (x - 1)^2$ in $0 < x < 1$.
 Hence show that i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$ ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$. [10M]